Lecture 10

2023/2024

Microwave Devices and Circuits for Radiocommunications

2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 16-18, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2023/2024

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 20.02.2024)
    - personal homework

Materials

- http://rf-opto.etti.tuiasi.ro

Site

English | Romana |

Main Courses Master Staff Research Students

Microwave and Optoelectronics Laboratory 

We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (**ETTI**) from the "Gh. Asachi" Technical University (**TUIASI**) in Iasi, Romania

We currently cover inside **ETTI** the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

Courses

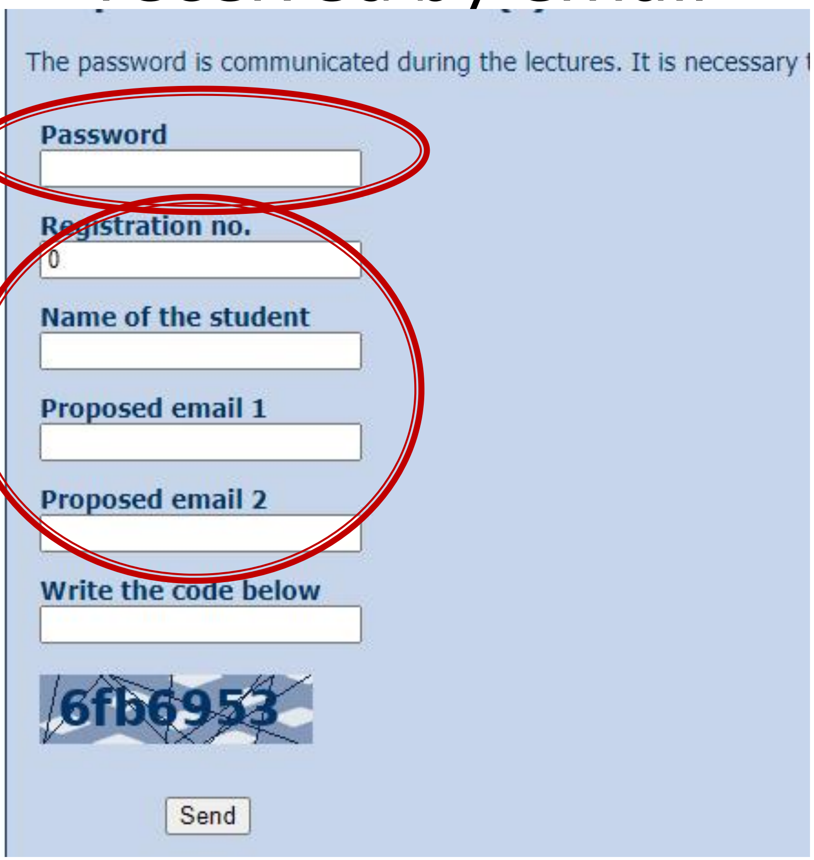
Nr.	Course	Shortcut	Code	Type	Semester	Credits	Weekly	Examination	Link
1	Microwave Devices and Circuits for Radiocommunications	**DCMR**	DOS412T	DOS	7	4	0P,1L,0S,2C	Exam	details
2	Monolithic Microwave Integrated Circuits	**CIMM**	RD.IA.207	DOMS	11	6	1.5L,0S,2C,0P	Exam	details
3	Advanced Techniques in the Design of the Radio-communications Systems	**TAPSR**	RD.IA.103	DIMS	9	6	1.5P,0L,0S,2C	Exam	details
4	Optical Communications	**CO**	DOS409T	DOS	7	5	0P,1L,0S,3C	Colloquium	details
5	Optical Communications	**OC**	EDOS409T	DOS	7	5	0P,1L,0S,3C	Exam	details
6	Satellite Communications	**CS**	RC.IA.104	DIMS	9	6	0L,0S,2C,1.5P	Exam	details
7	Applied Informatics 1	**IA1**	DOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
8	Applied Informatics 1	**AI1**	EDOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
9	Databases, Web Programming and Interfacing	**DWPI**	ITT.IA.601	DIS	11	5	1P,1L,0.25S,1C	Verification	details
10	Web Applications Design	**PAW**	RC.IA.108	DIMS	10	5	1L,0S,1.5C,1P	Exam	details
11	Optoelectronics	**OPTO**	DID405M	DID	8	4	0P,1L,0S,2C	Colloquium	details
12	Microwave Devices and Circuits for Radiocommunications (English)	**MDCR**	EDOS412T	DOS	8	4	0P,1L,0S,2C	Exam	details

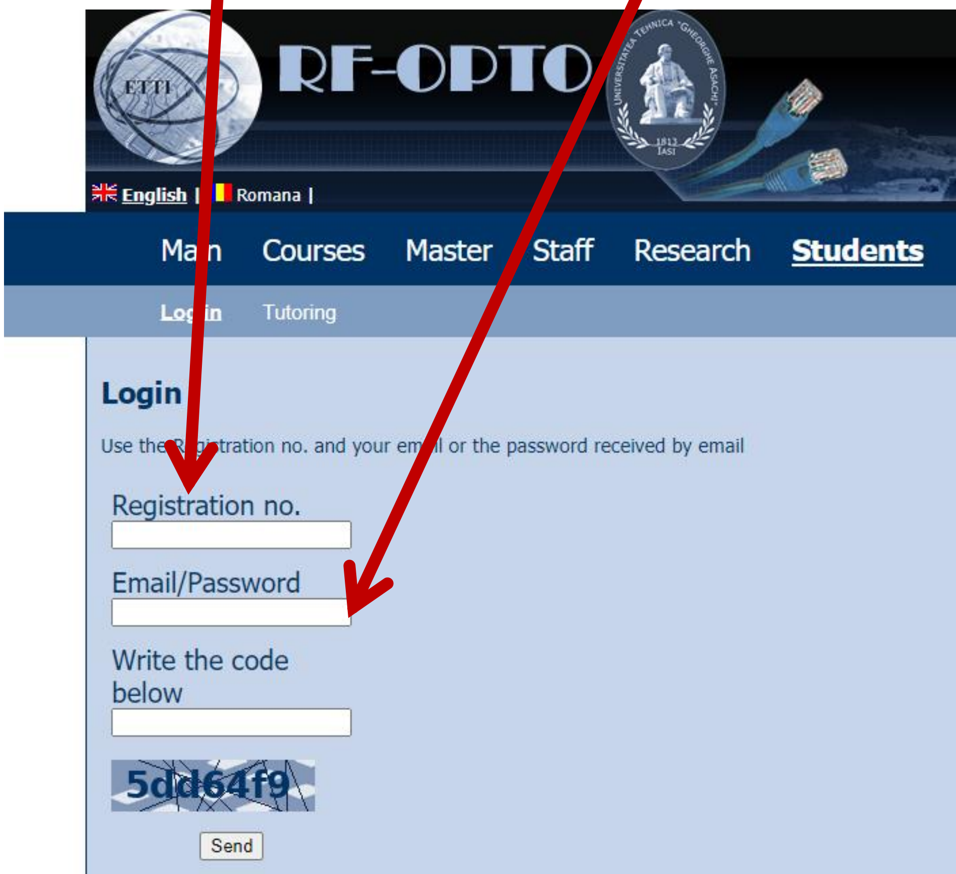
Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/**online exam > Week4-Week6**
  - used at lectures/laboratory

Online – Registration no.

- access to **online exams** requires the **password** received by email





Password

- received by email

Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

____

Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply | Reply all | Forward

Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020	[redacted]
[redacted]	[redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

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Password: [redacted]

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Save this message in a safe place for later use

Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~

Materials

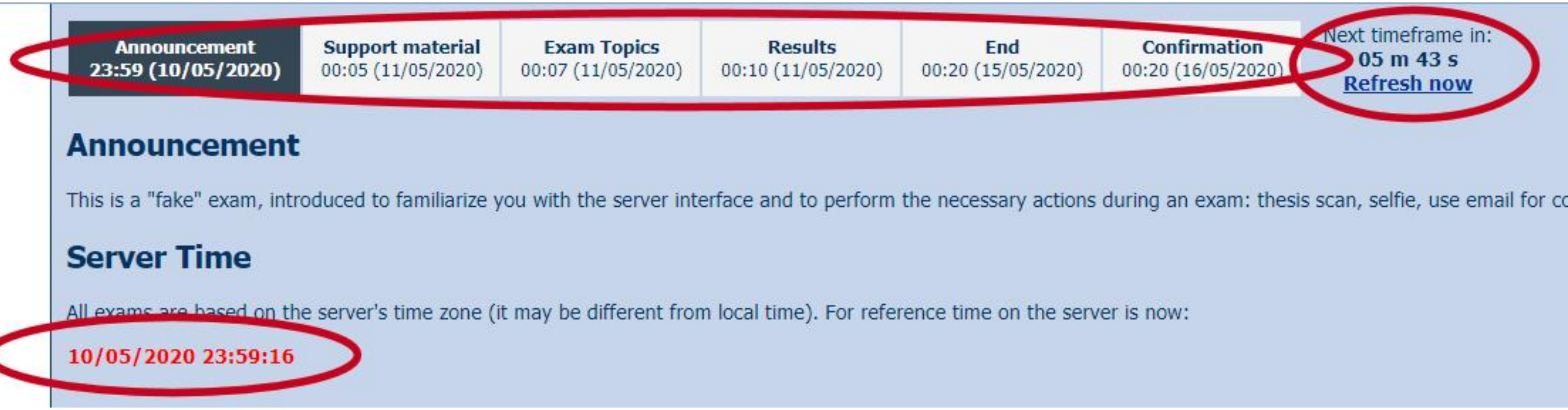
Other data

Manual examen on-line (pdf, 2.65 MB, ro,)
Simulare Examen (video) (mp4, 65.12 MB, ro,)

Microwave Devices and Circuits (Englis

Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri s	T2, fisier parmetri s	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

√

✗

Online results submission

Grade = Quality of the work +

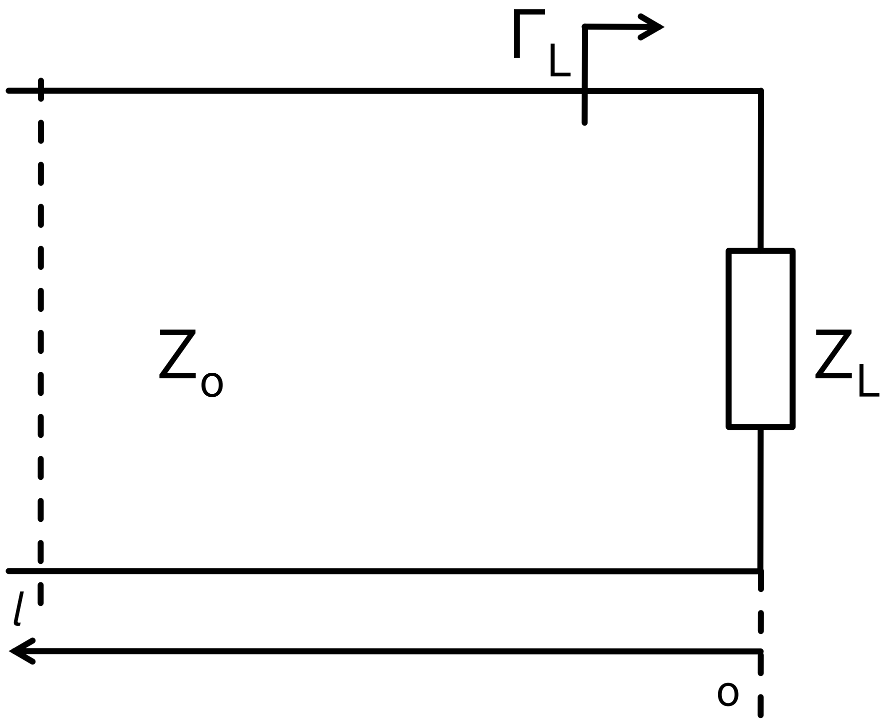
\+ Quality of the submission

TEM transmission lines

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_o real

The lossless line

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z)=\frac{V_0^+}{Z_0}\cdot\left(e^{-j\cdot\beta\cdot z}-\Gamma\cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

$$P_{avg}=\frac{1}{2}\cdot\mathrm{Re}\left\{V(z)\cdot I(z)^*\right\}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\mathrm{Re}\left\{1-\Gamma^*\cdot e^{-2j\cdot\beta\cdot z}+\Gamma\cdot e^{2j\cdot\beta\cdot z}-\left|\Gamma\right|^2\right\}$$

$$\left(z-z^*\right)=\mathrm{Im}$$

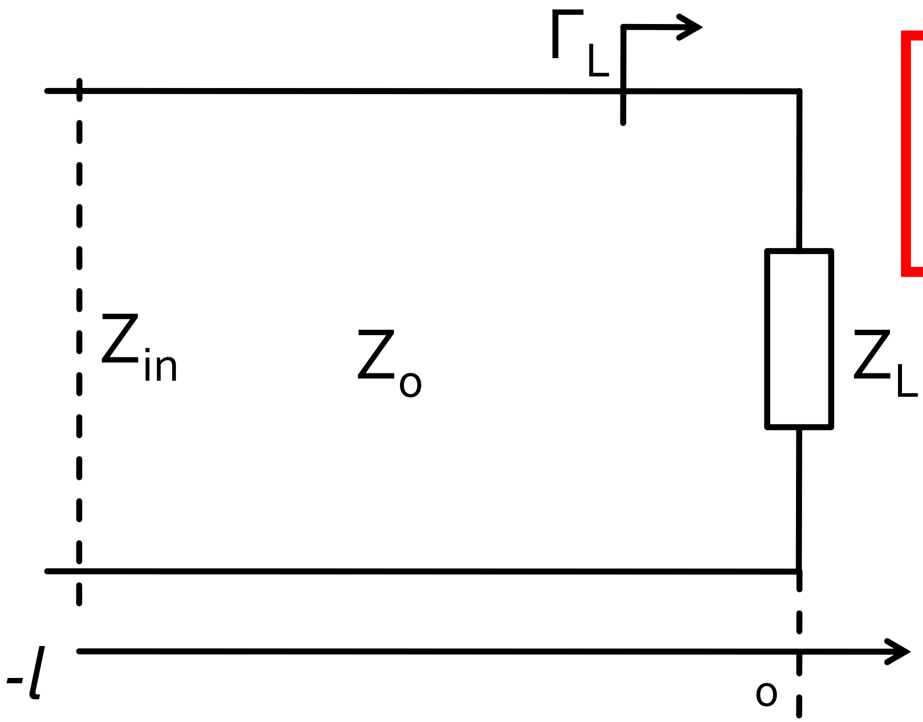
$$P_{avg}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\left(1-\left|\Gamma\right|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB]

$$\mathrm{RL}=-20\cdot\log\left|\Gamma\right| \quad [\mathrm{dB}]$$

The lossless line

- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$

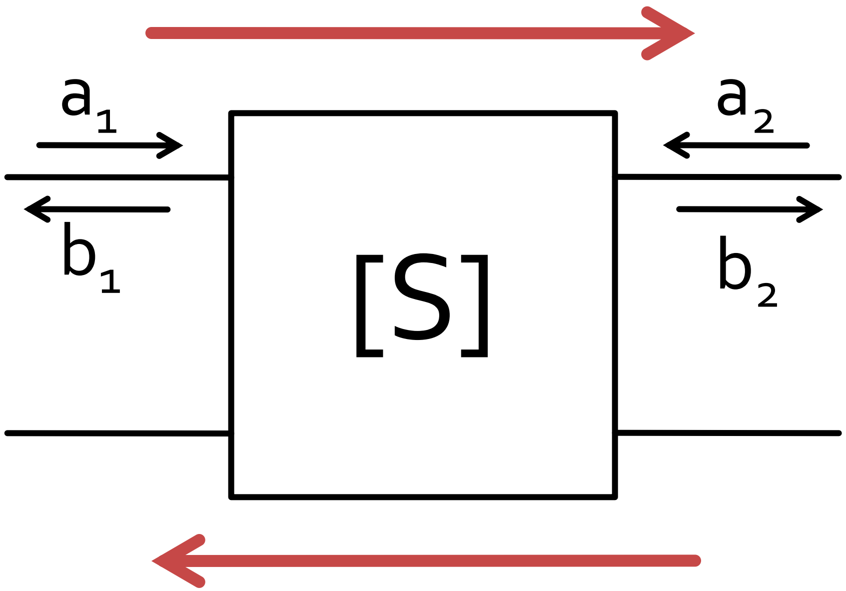


$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

General theory

Microwave Network Analysis

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

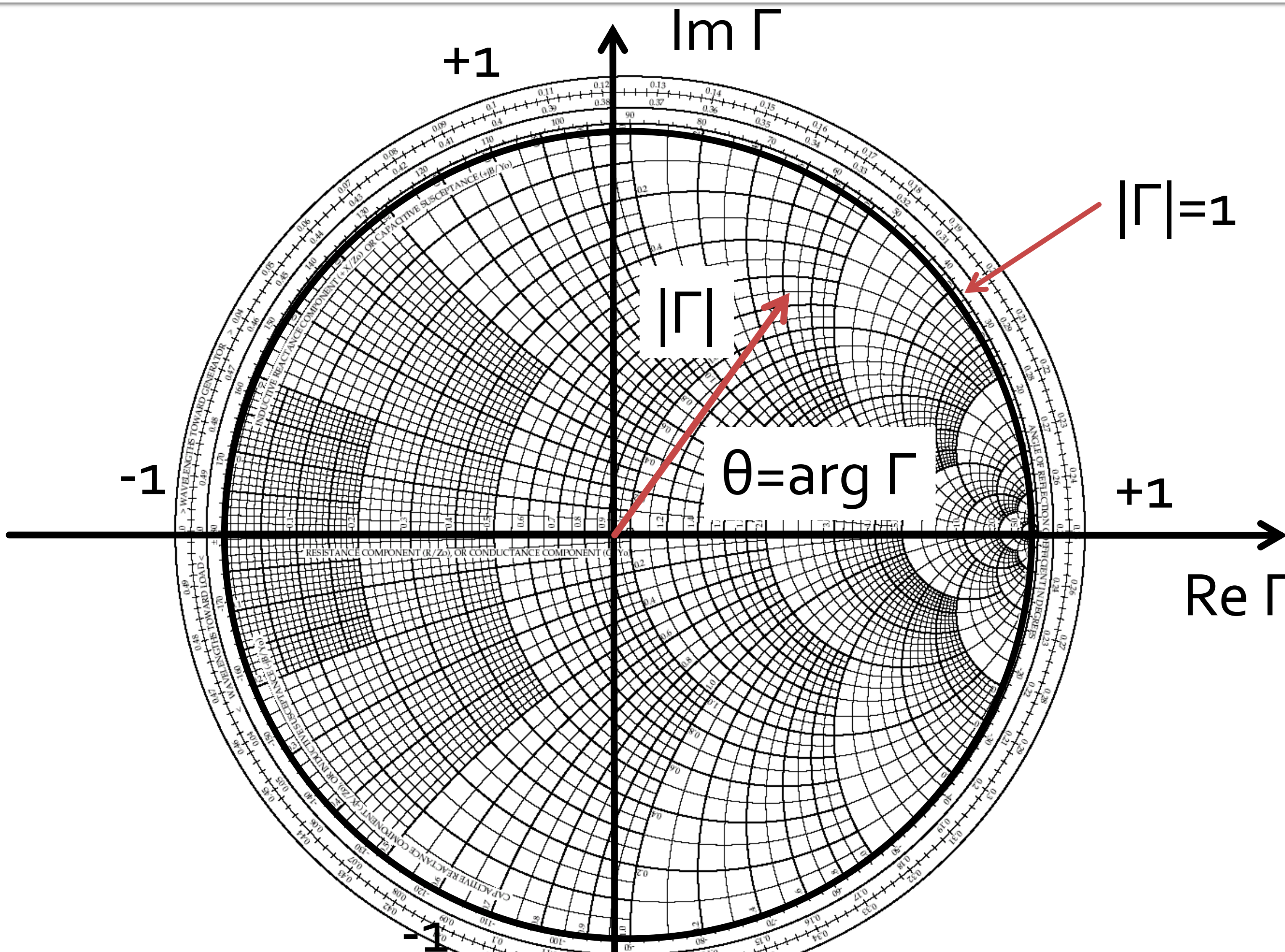
$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

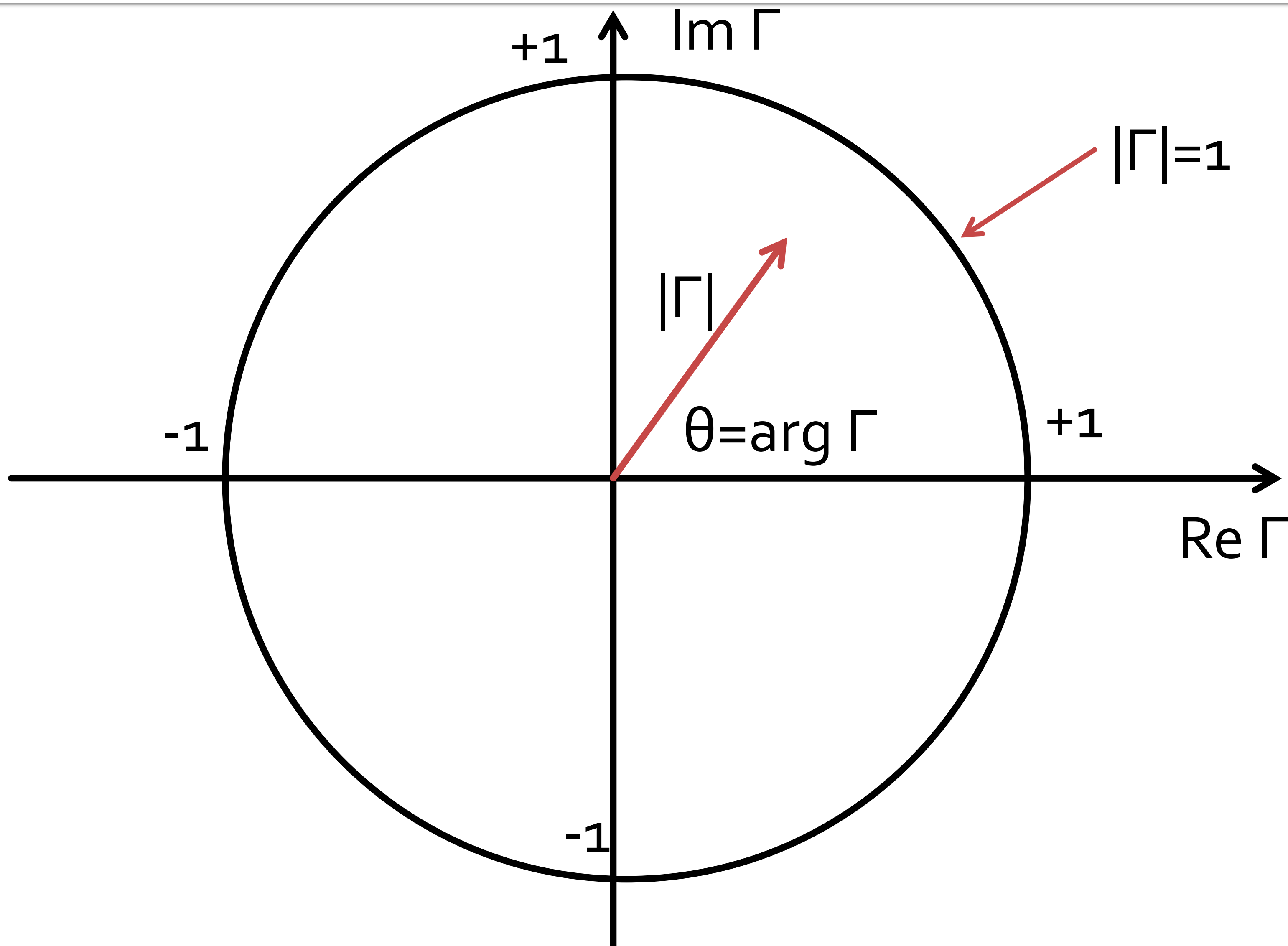
Impedance Matching

The Smith Chart

The Smith Chart



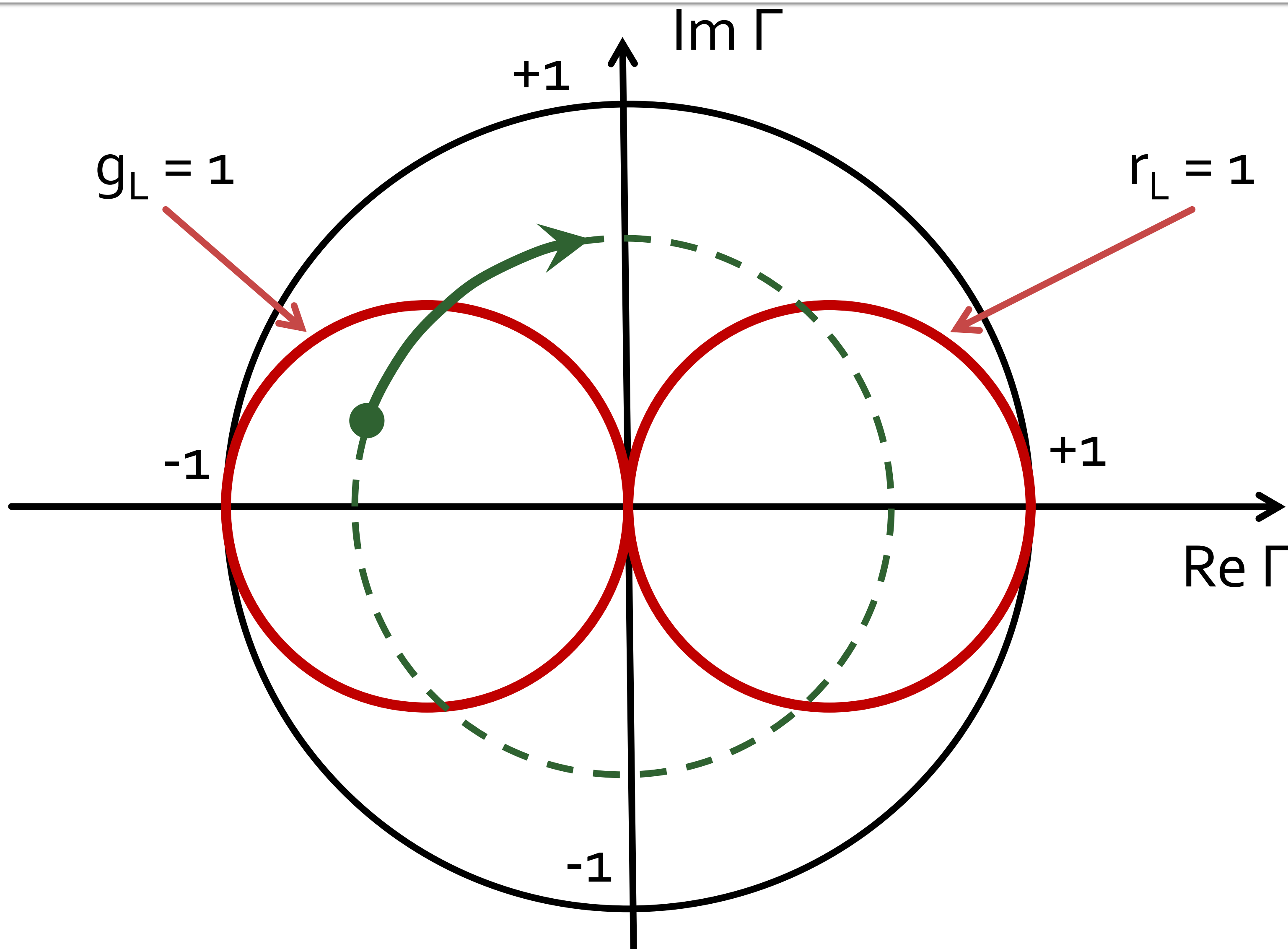
The Smith Chart



Impedance Matching

Impedance Matching with Stubs

Smith chart, r=1 and g=1

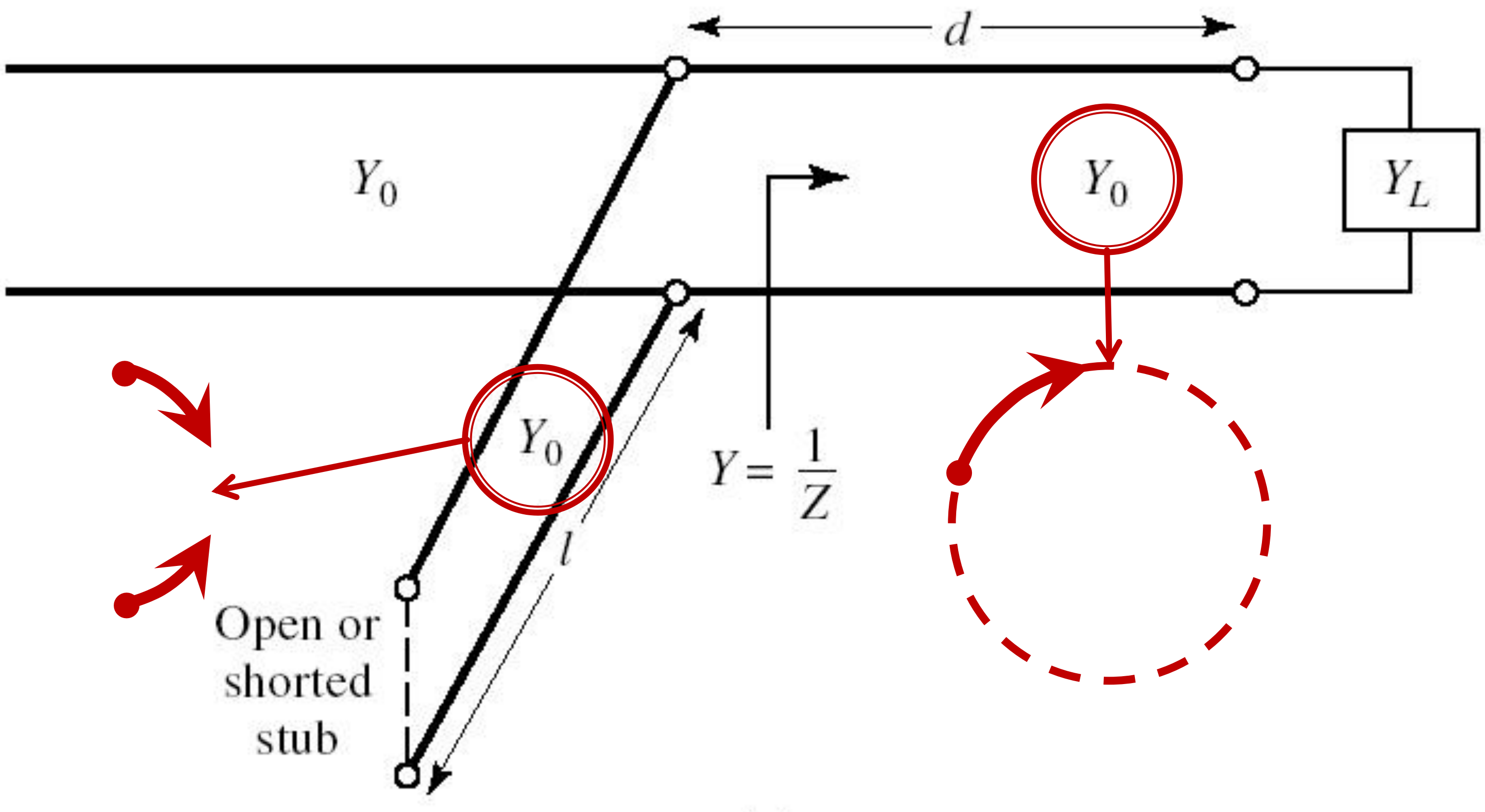


Analytical solutions

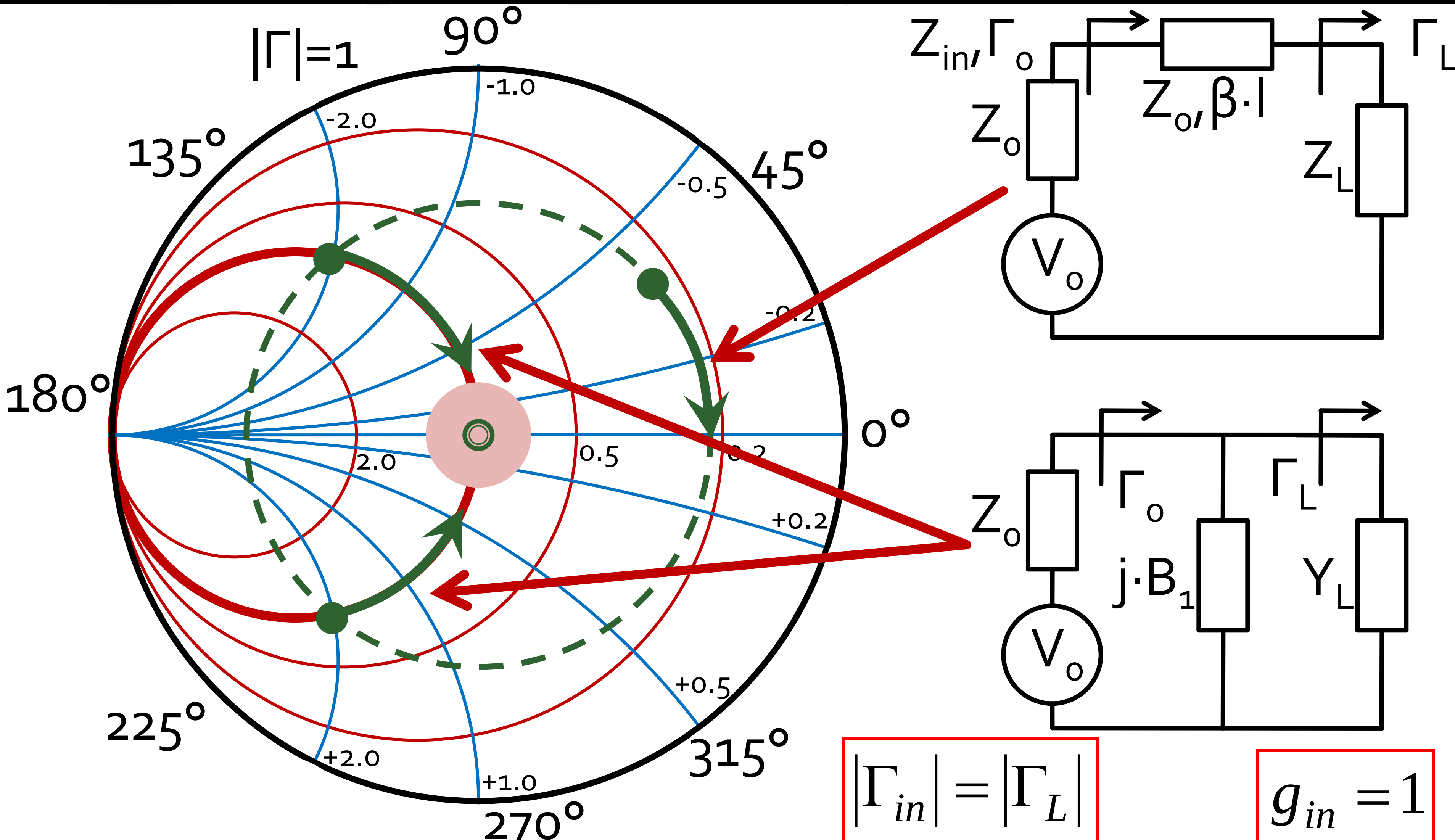
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S| \qquad \theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$
  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

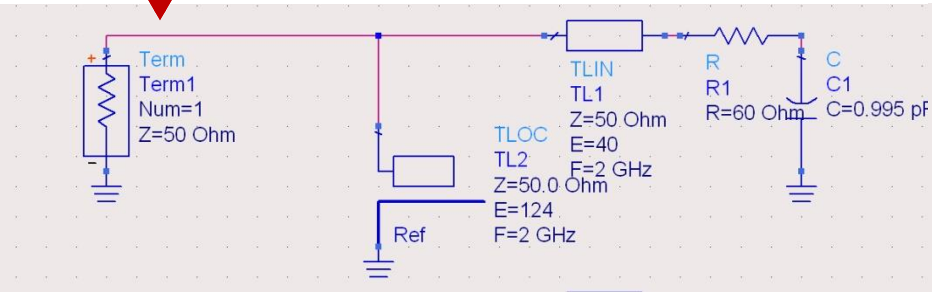
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

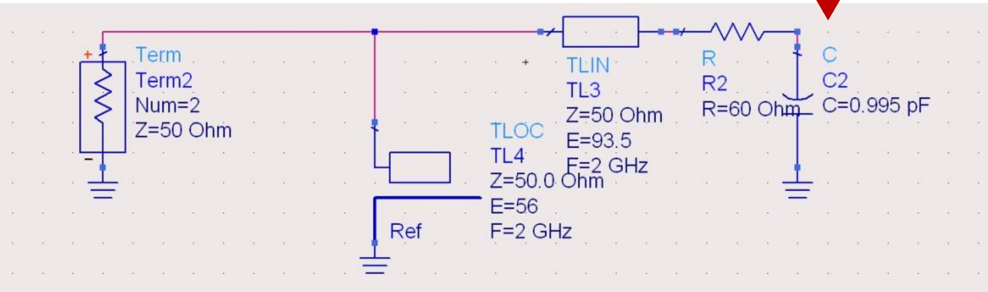
$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

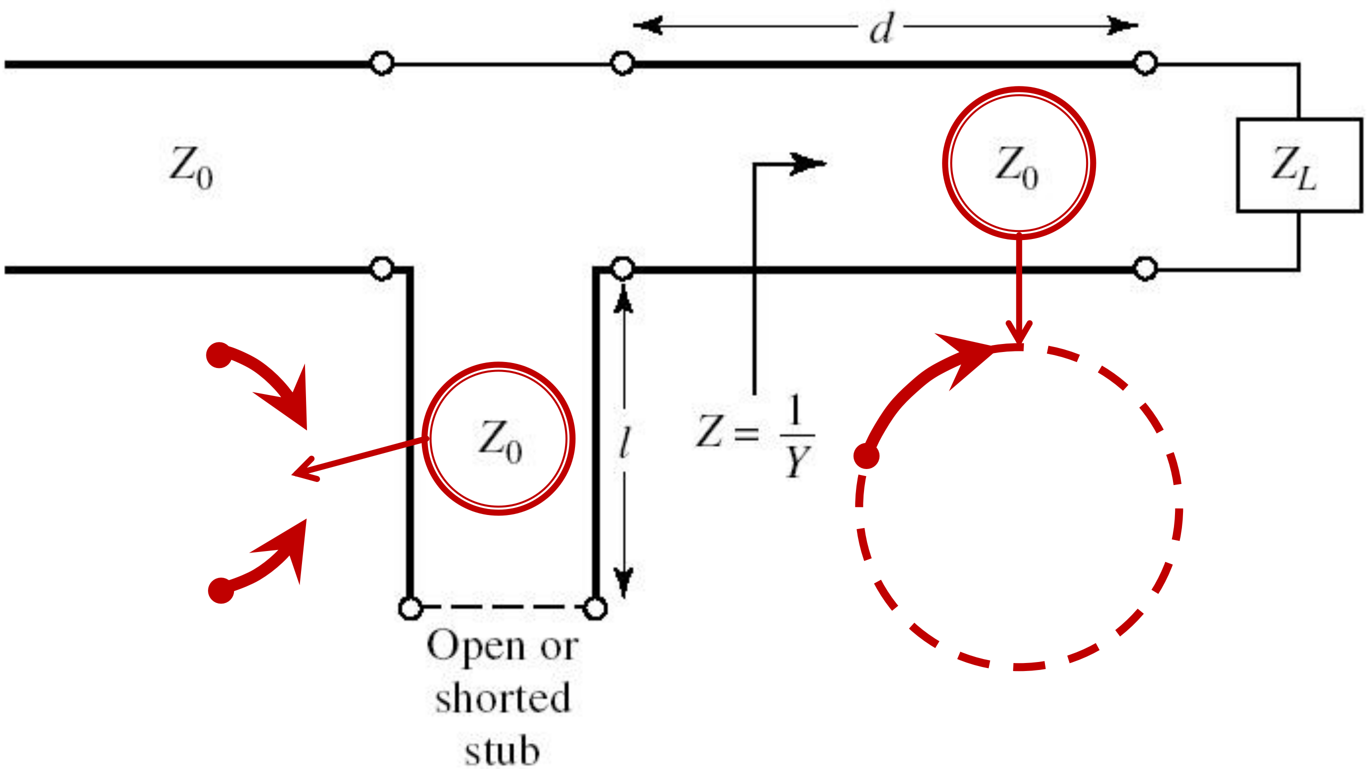
$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$



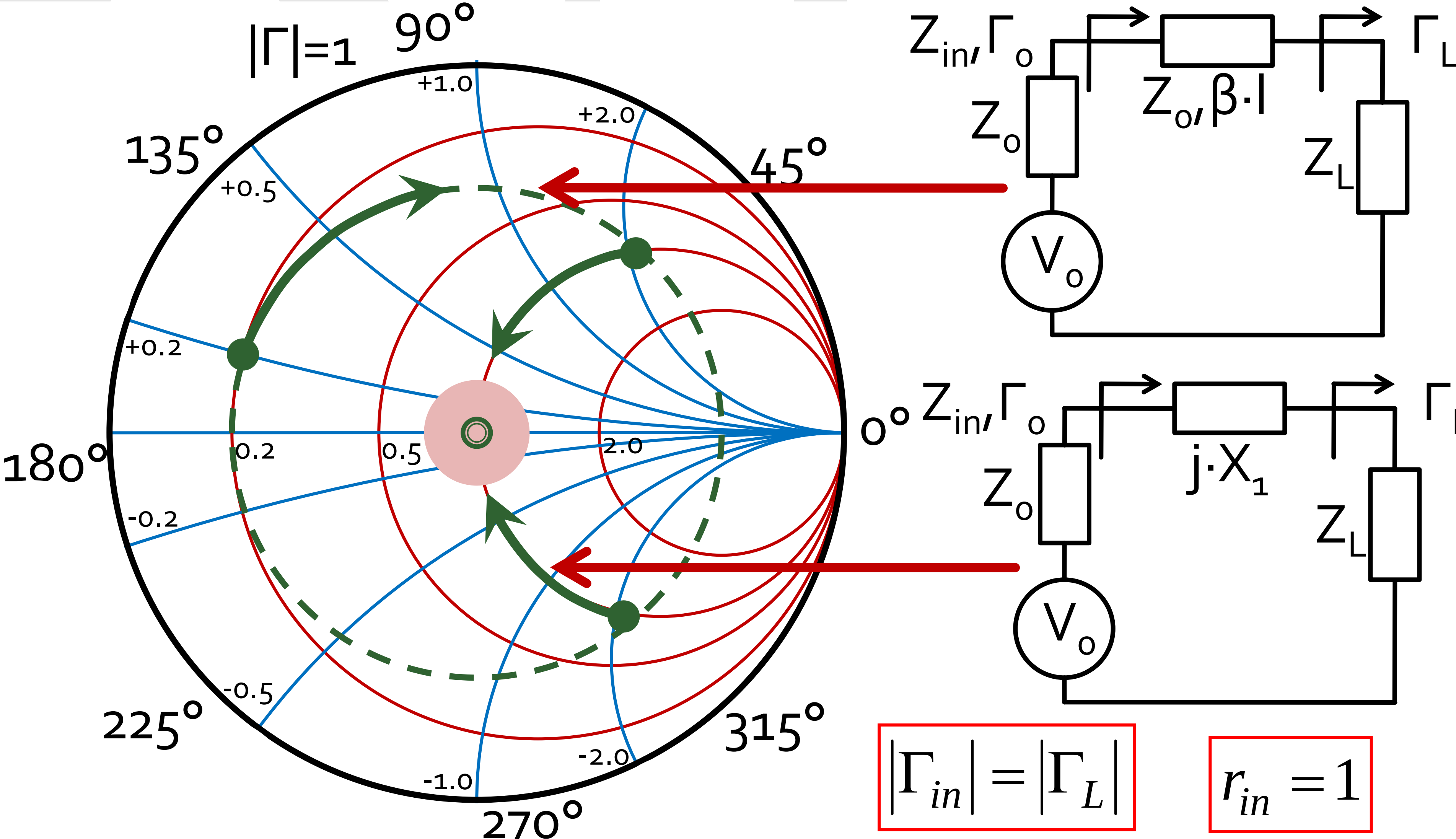


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8° (+180°) \rightarrow \theta_{ss} = 143.2°$$

  - **"-" solution**

$$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2° (+180°) \rightarrow \theta = 166.8°$$

$$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8°$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+56.28^\circ\\-56.28^\circ\end{cases}\quad \theta=\begin{cases}43.1^\circ\\166.8^\circ\end{cases}\quad \mathrm{Im}[z_S(\theta)]=\begin{cases}+1.335\\-1.335\end{cases}\quad \theta_{ss}=\begin{cases}-36.8^\circ+180^\circ=143.2^\circ\\+36.8^\circ\end{cases}$$

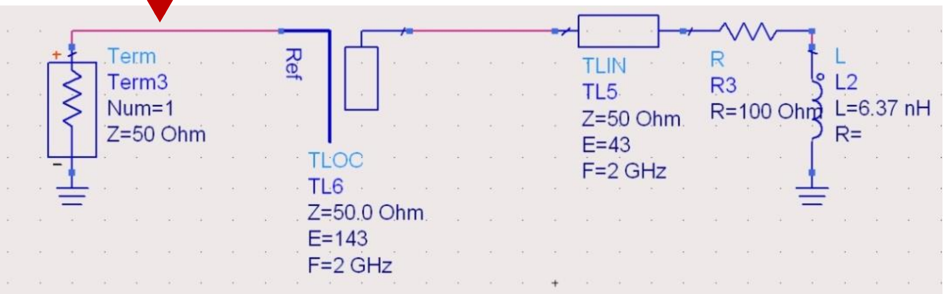
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

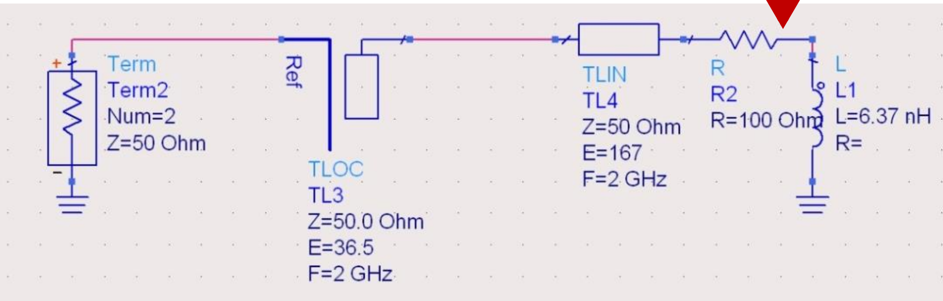
$$l_1=\frac{43.1^\circ}{360^\circ}\cdot\lambda=0.120\cdot\lambda$$

$$l_2=\frac{143.2^\circ}{360^\circ}\cdot\lambda=0.398\cdot\lambda$$

$$l_1=\frac{166.8^\circ}{360^\circ}\cdot\lambda=0.463\cdot\lambda$$

$$l_2=\frac{36.8^\circ}{360^\circ}\cdot\lambda=0.102\cdot\lambda$$





Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

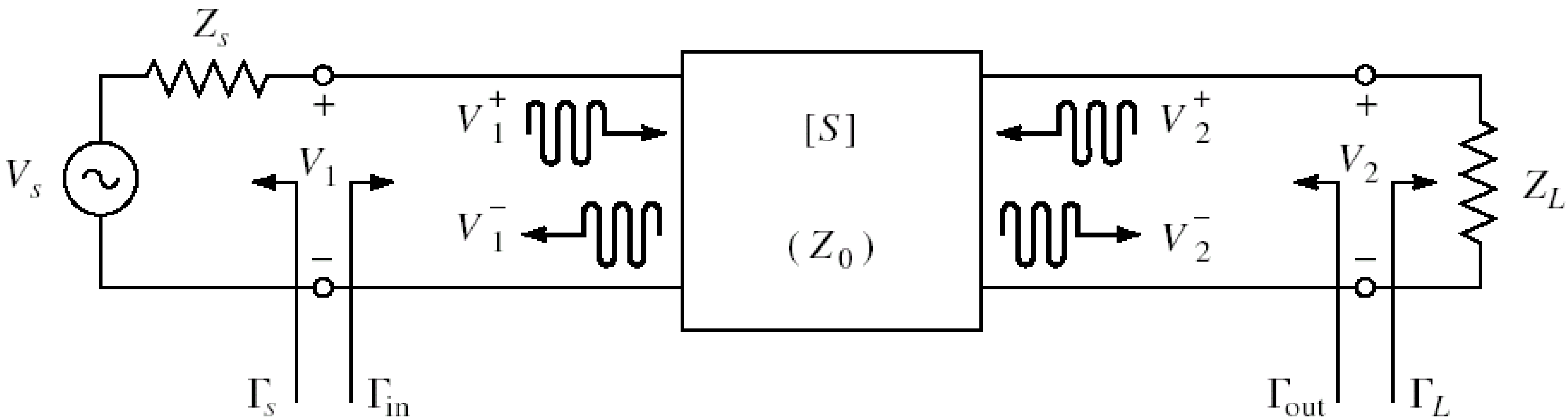
  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

Microwave Amplifiers

Amplifier as two-port



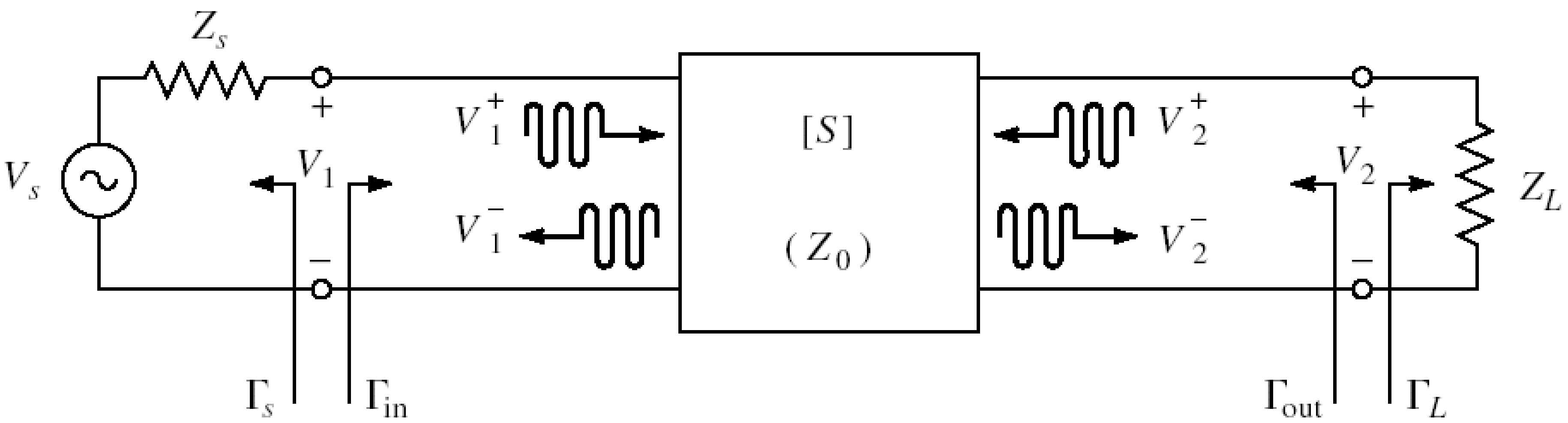
- Charaterized with S parameters
- normalized at Zo (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

S2P - Touchstone

- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V     ID = 15 mA
# GHz  S  MA  R  50
!  f        S11          S21          S12          S22
! GHz     MAG   ANG      MAG   ANG      MAG   ANG      MAG   ANG
 1.000 0.9800  -18.0   2.230  157.0  0.0240   74.0  0.6900  -15.0
 2.000 0.9500  -39.0   2.220  136.0  0.0450   57.0  0.6600  -30.0
 3.000 0.8900  -64.0   2.210  110.0  0.0680   40.0  0.6100  -45.0
 4.000 0.8200  -89.0   2.230   86.0  0.0850   23.0  0.5600  -62.0
 5.000 0.7400 -115.0   2.190   61.0  0.0990    7.0  0.4900  -80.0
 6.000 0.6500 -142.0   2.110   36.0  0.1070  -10.0  0.4100  -98.0
!
!  f      Fmin   Gammaopt  rn/50
! GHz      dB    MAG  ANG    -
 2.000     1.00  0.72   27   0.84
 4.000     1.40  0.64   61   0.58
```

Amplifier as two-port

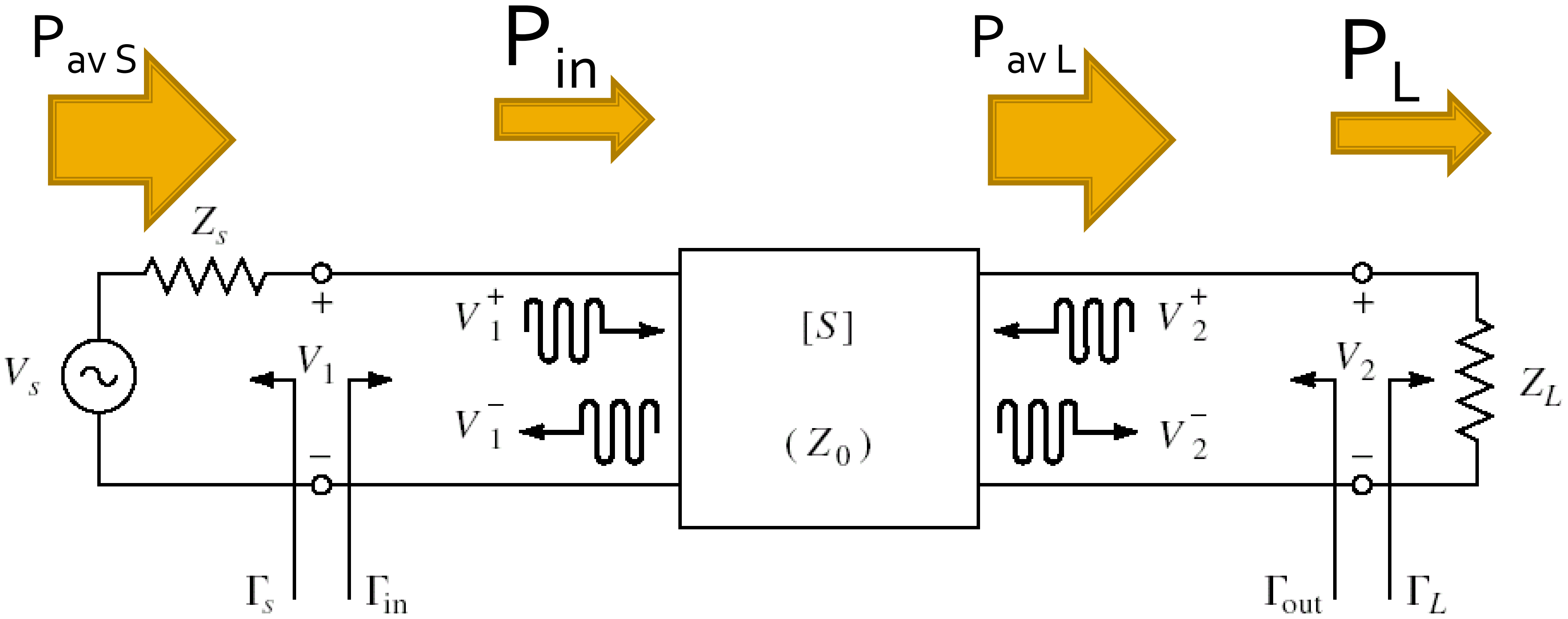


$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Power / Matching

- Two ports in which matching influences the power transfer



Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

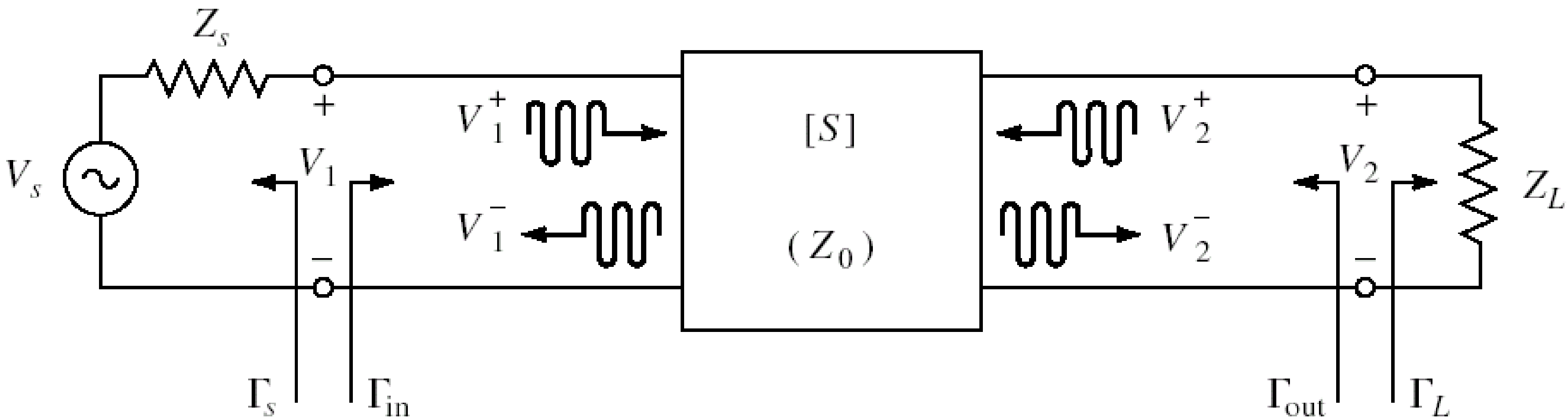
$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

Microwave Amplifiers

Stability

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Stability

- L7 $\Gamma = \Gamma_r + j \cdot \Gamma_i$ $r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$

  Z_{in} $\Gamma_{in} = \Gamma_r + j \cdot \Gamma_i$

- instability

  $\mathrm{Re}\{Z_{in}\} < 0 \quad \Leftrightarrow \quad 1-\Gamma_r^2-\Gamma_i^2 < 0$ $\Gamma_r^2+\Gamma_i^2 > 1$ $|\Gamma_{in}| > 1$

- stability, Z_{in}
  - conditions to be met by Γ_L to achieve (input) stability

    $|\Gamma_{in}| < 1$ $\left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1-S_{22} \cdot \Gamma_L} \right| < 1$

- similarly Z_{out}
  - conditions to be met by Γ_S to achieve (output) stability

Output stability circle (CSOUT)

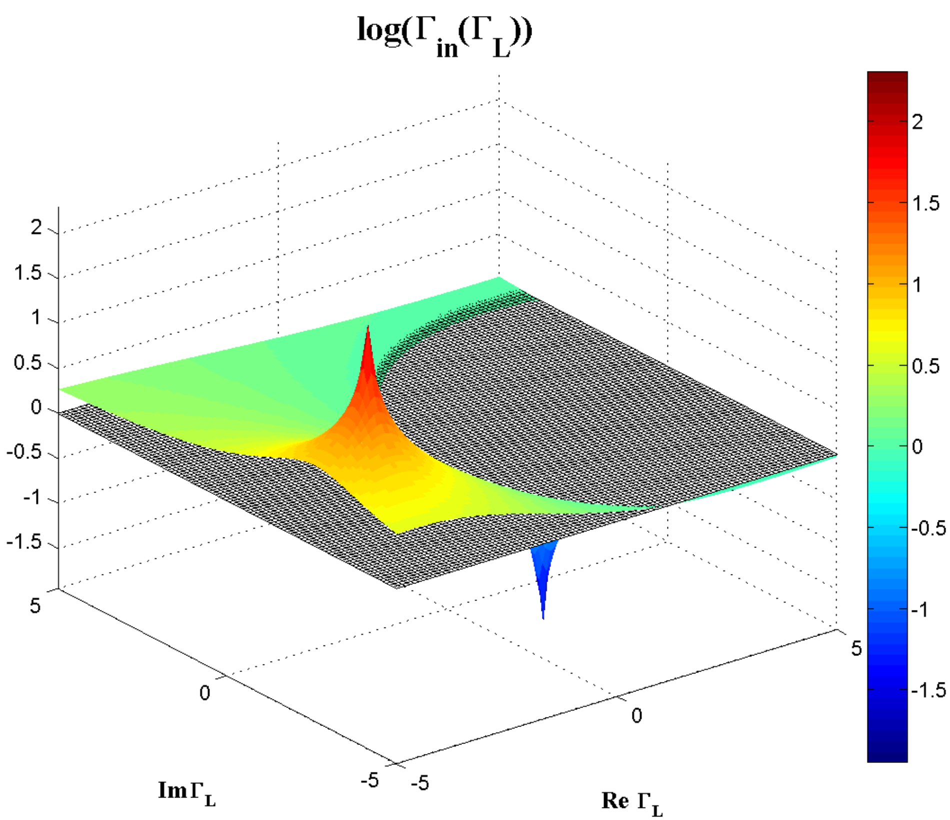
$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right| \qquad \left|\Gamma_L - C_L\right| = R_L$$

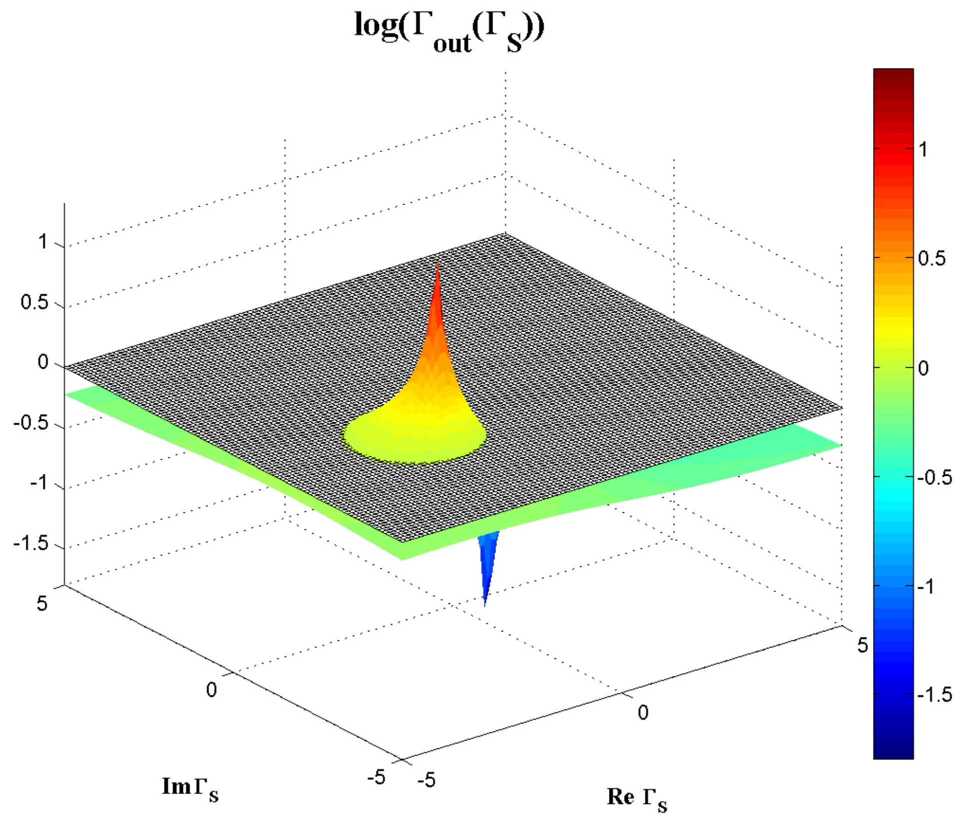
- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \qquad R_L = \frac{\left|S_{12} \cdot S_{21}\right|}{\left|\left|S_{22}\right|^2 - \left|\Delta\right|^2\right|}$$

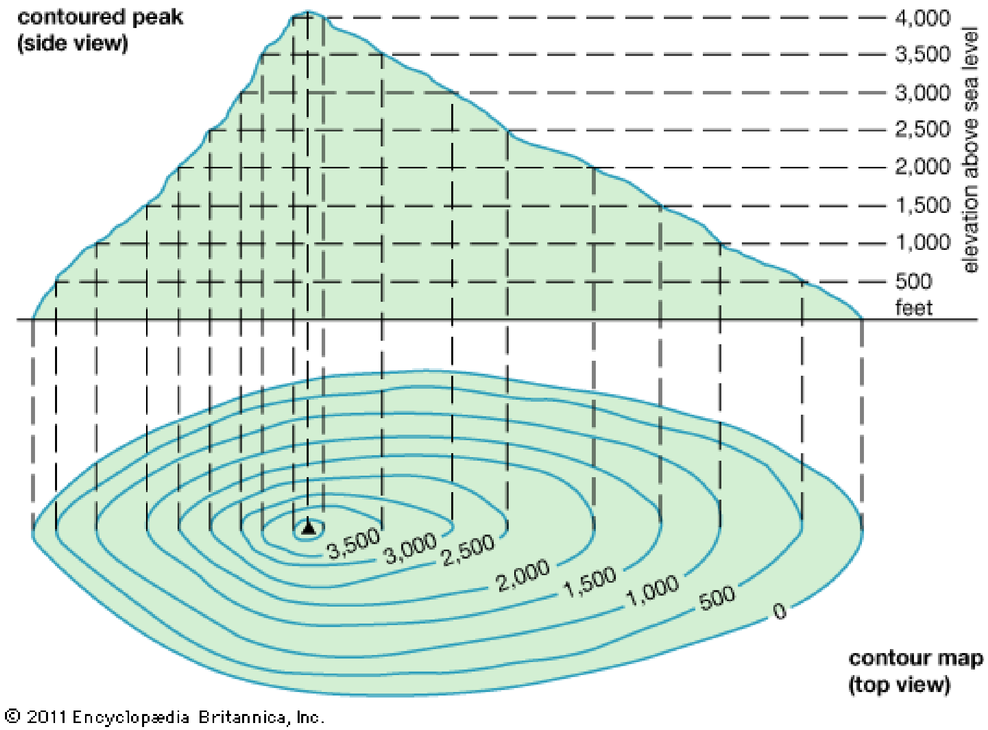
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

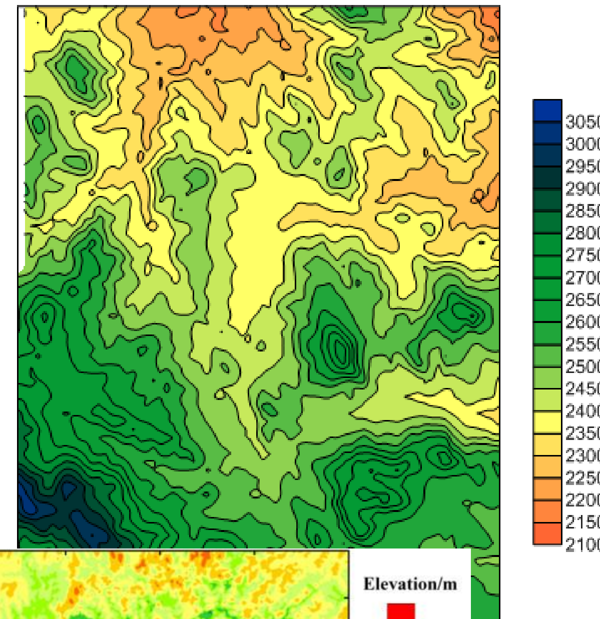
- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane z = 0 is a circle

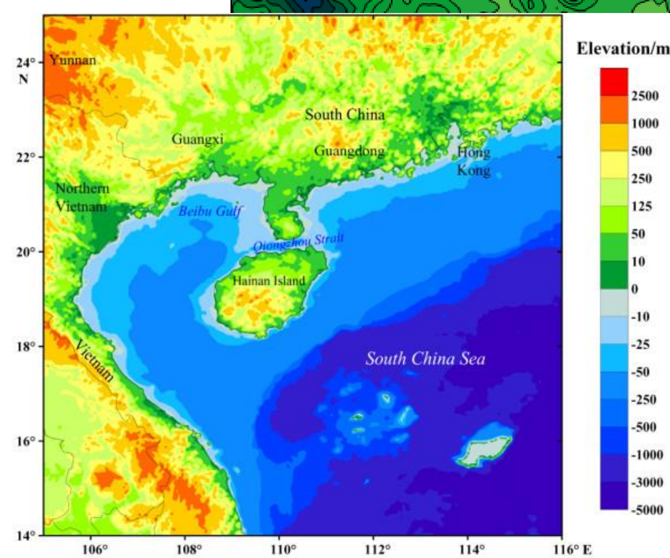




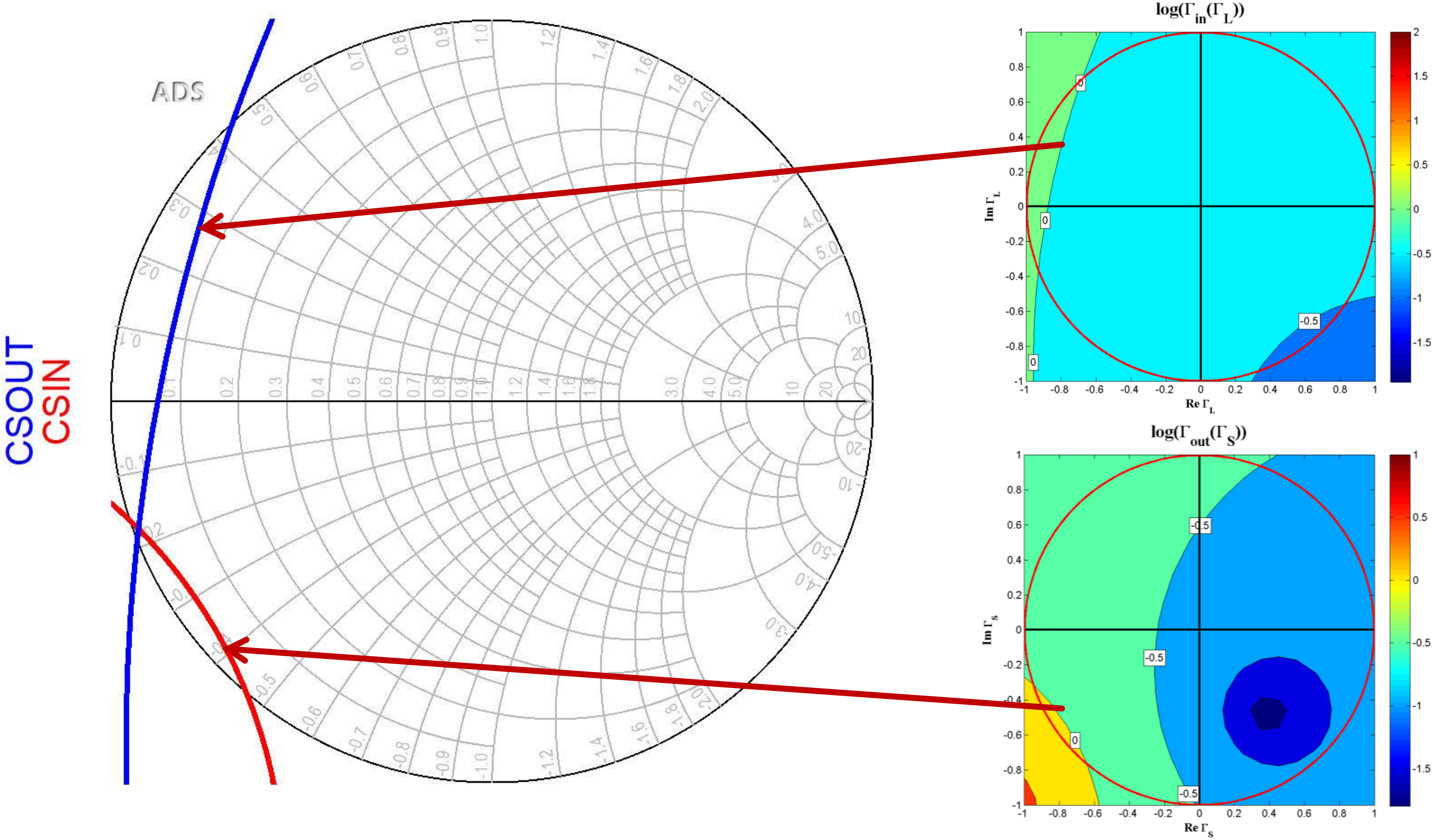
Contour map/lines



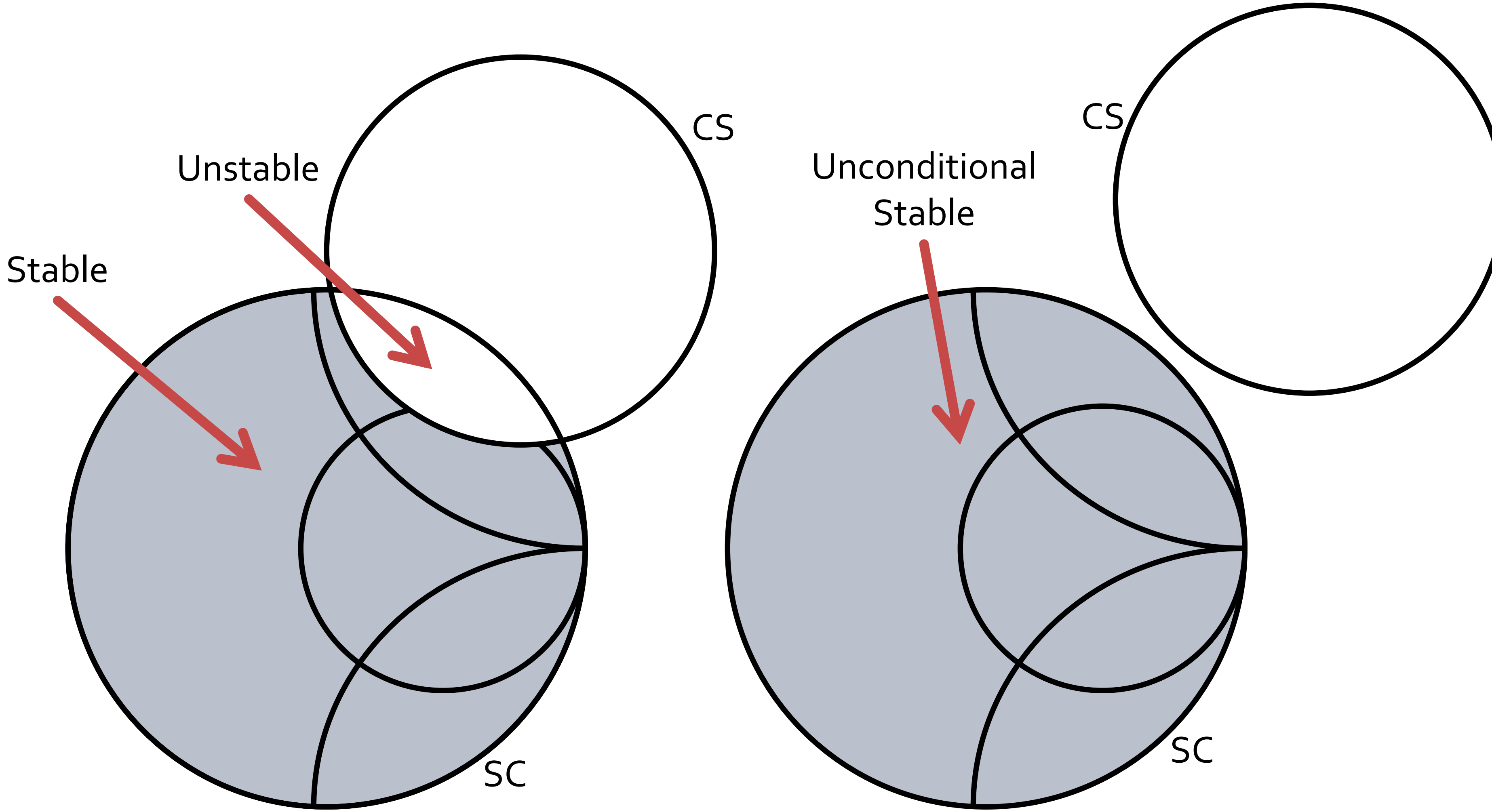




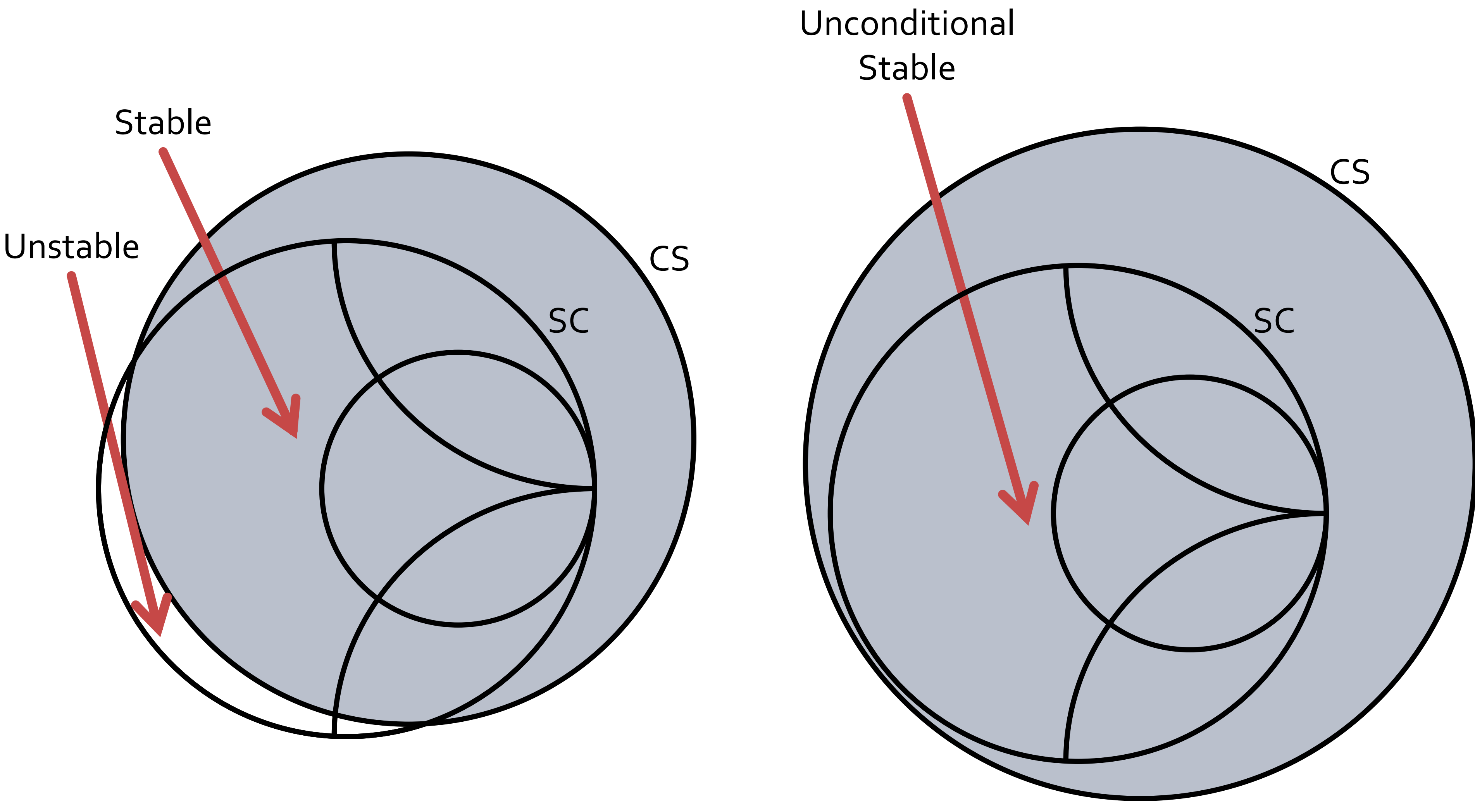
CSIN, CSOUT



Several possible positioning



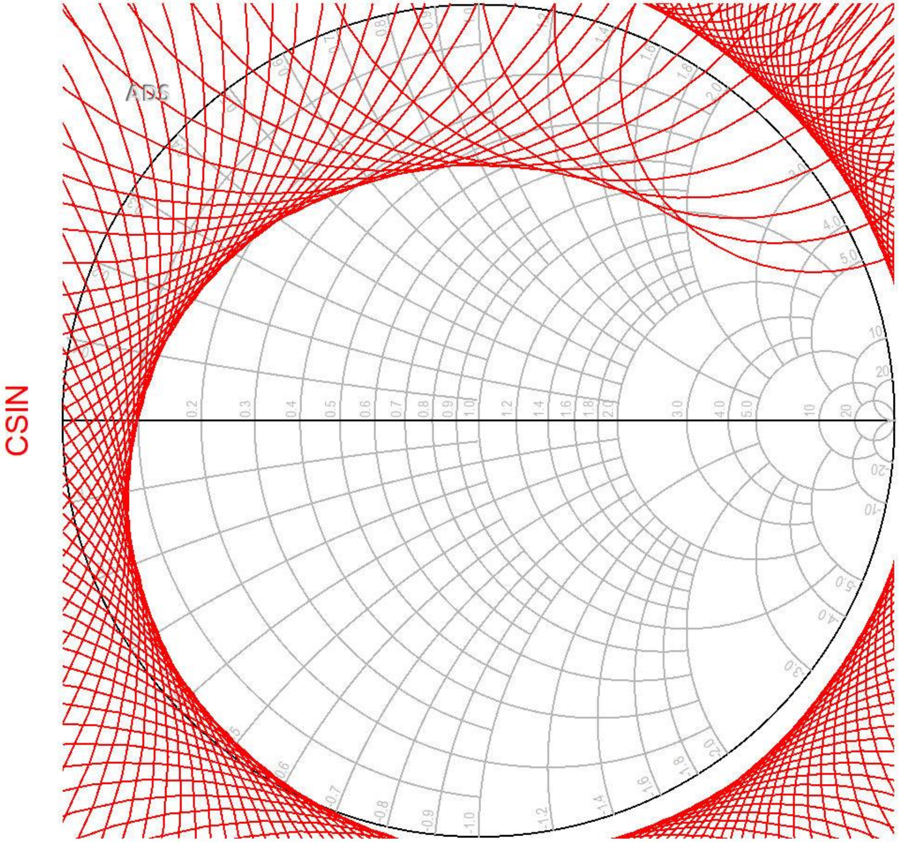
Several possible positioning

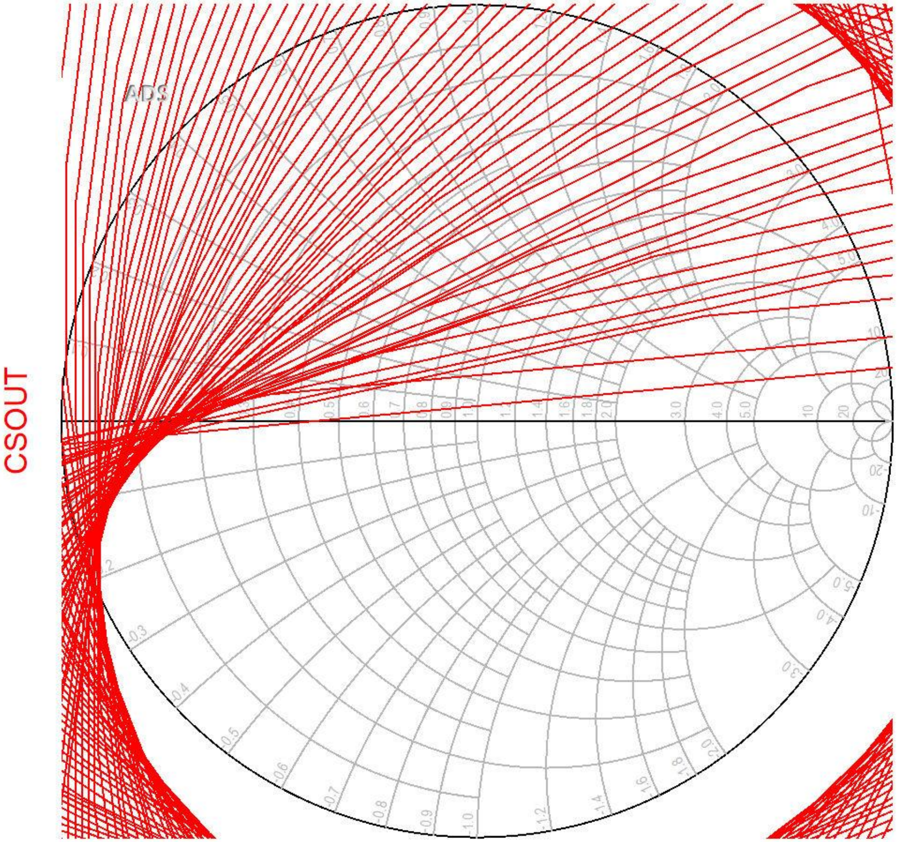


Stability

- **Unconditional stability:** the circuit is unconditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ for **any** passive impedance of the load/source
- **Conditional stability:** the circuit is conditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ only for **some** passive impedance of the load/source
  - passive impedance of the load/source <-> interior of the Smith Chart (radius 1 circle in the complex plane)

Circles in wide frequency range





Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|}$$

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

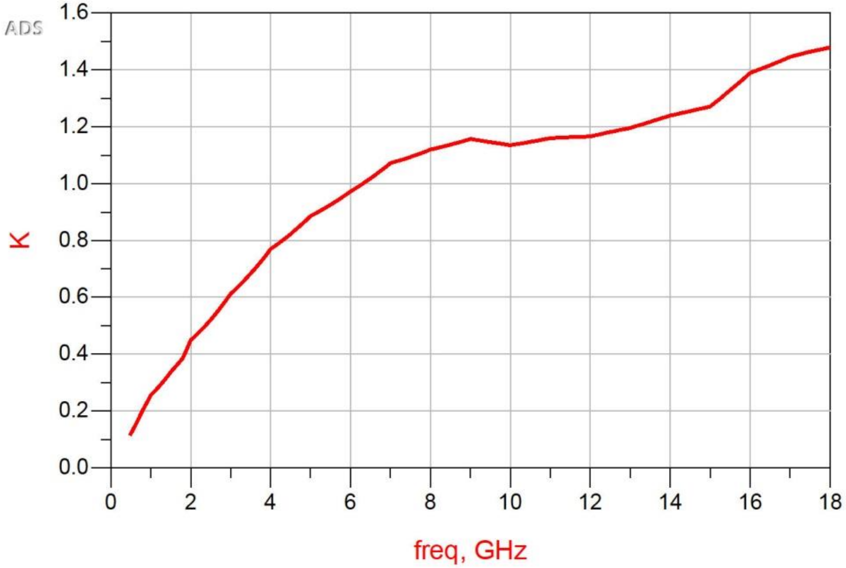
- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
  - $K > 1$
  - $|\Delta| < 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$

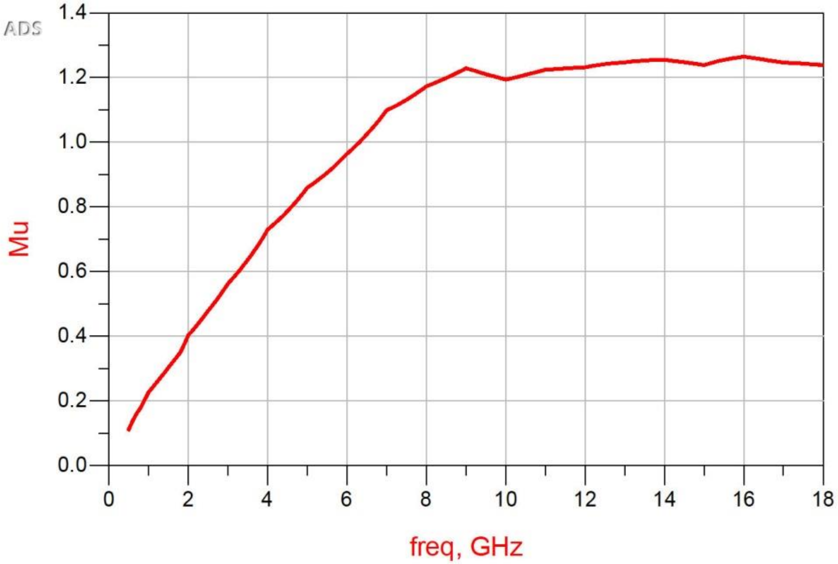
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1$$

$$|\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

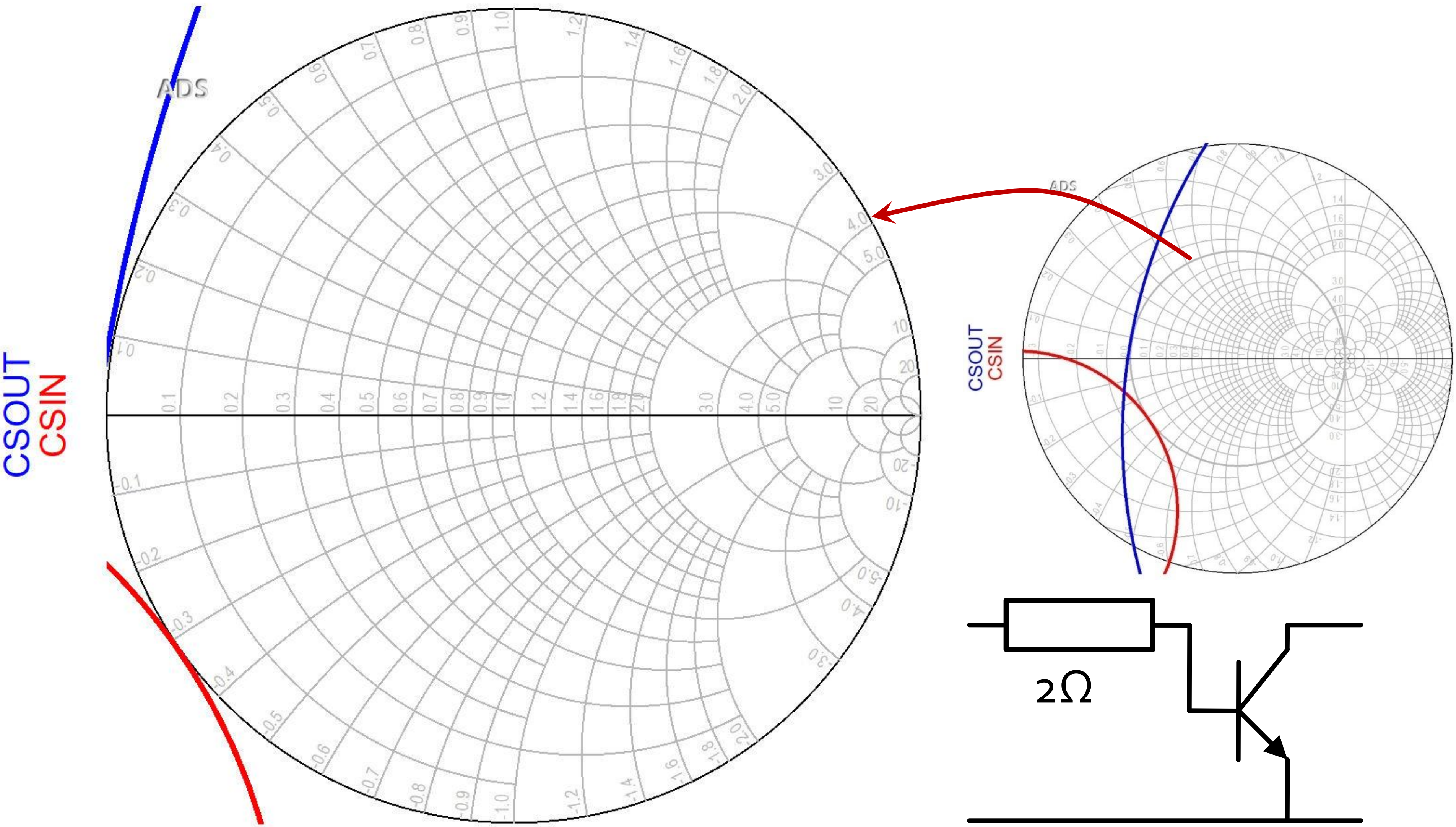
Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz
- unconditionally stable for f > 6.31GHz

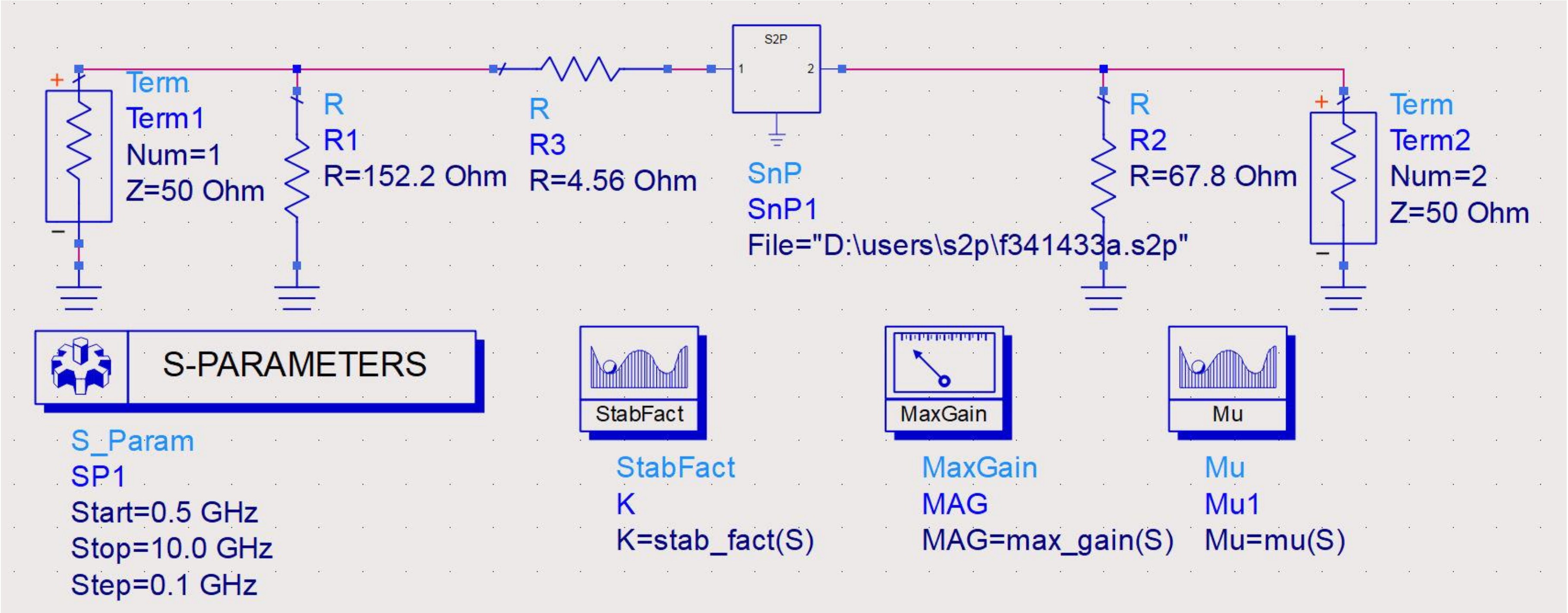




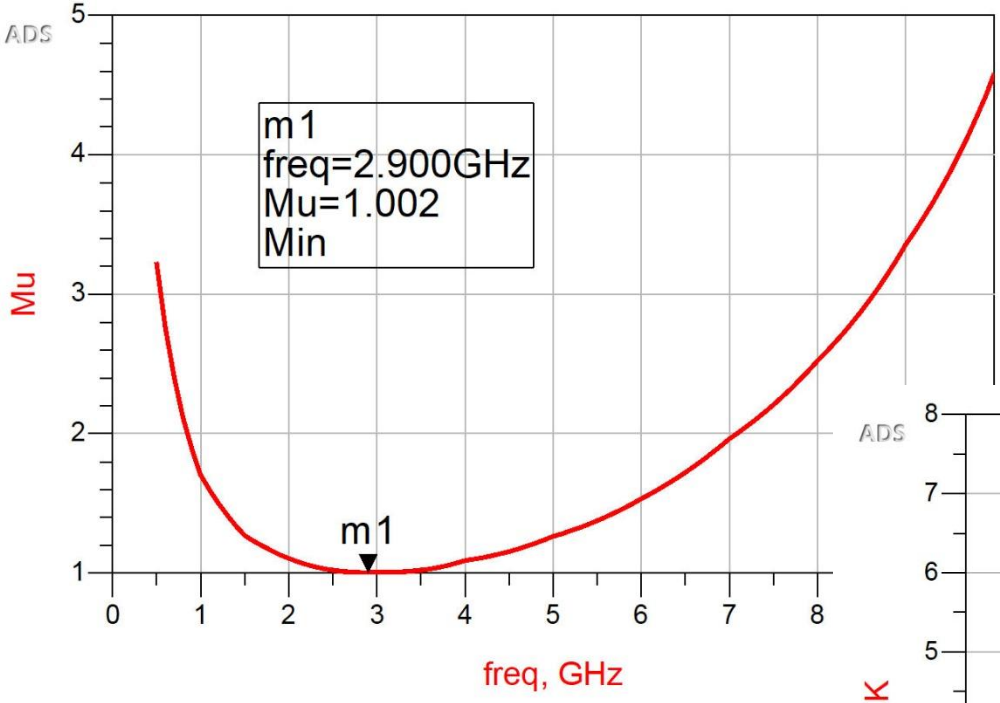
ADS, Rs = 2Ω

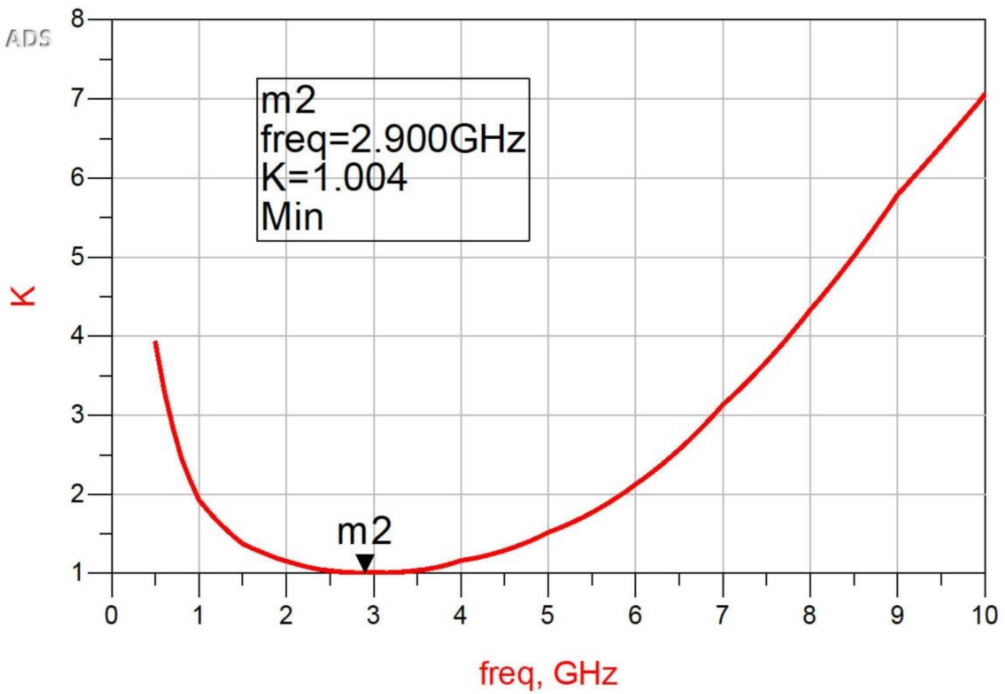


Stabilization of two-port

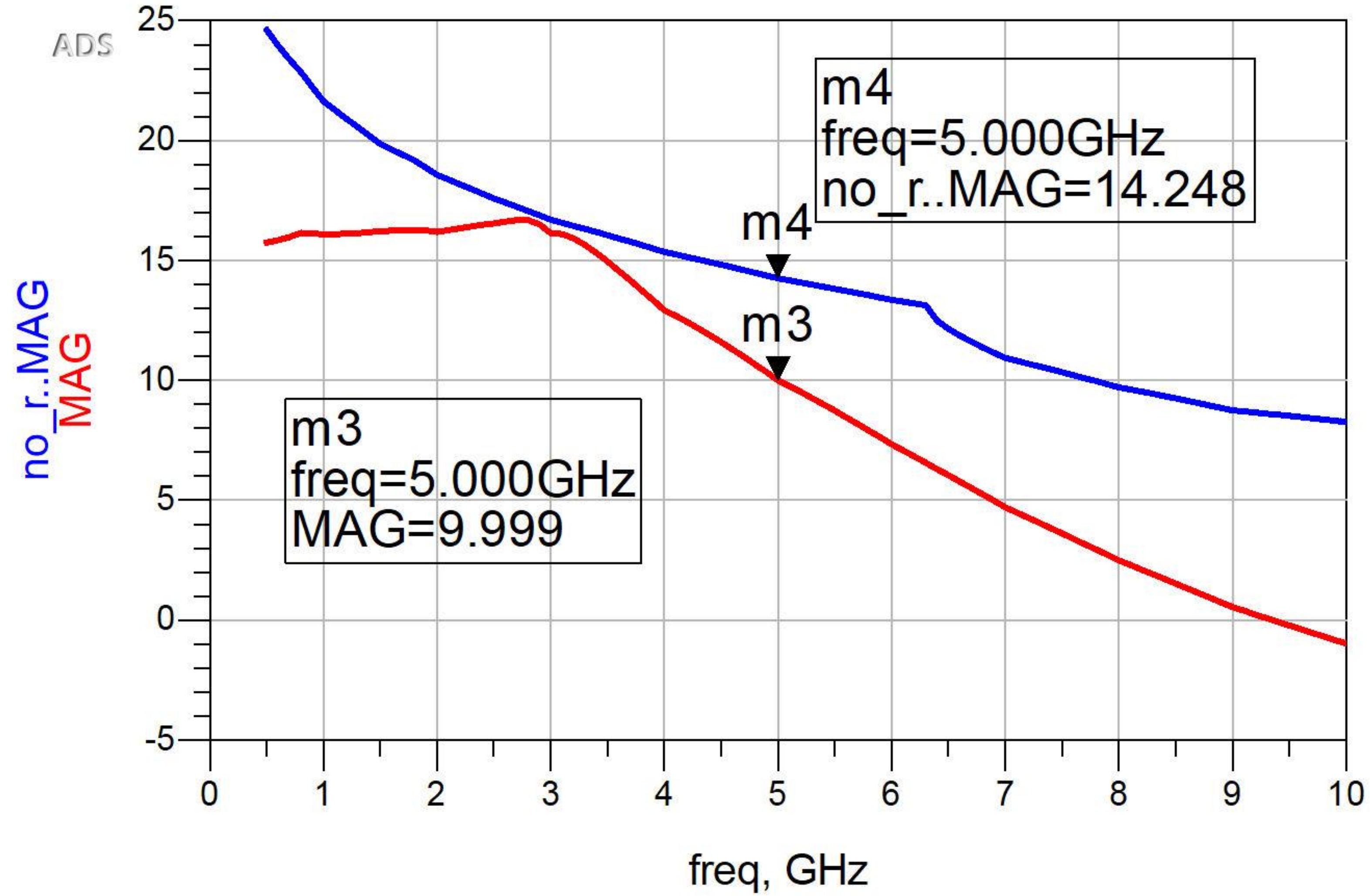


Stabilization of two-port





Stabilization of two-port

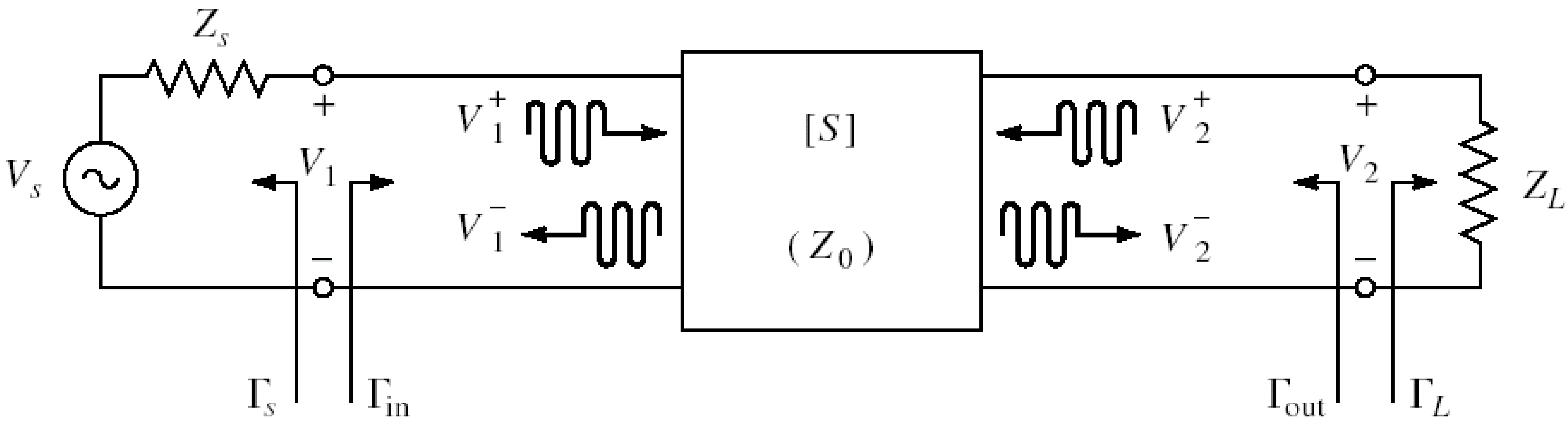


Continue

Microwave Amplifiers

Power Gain of Microwave Amplifiers

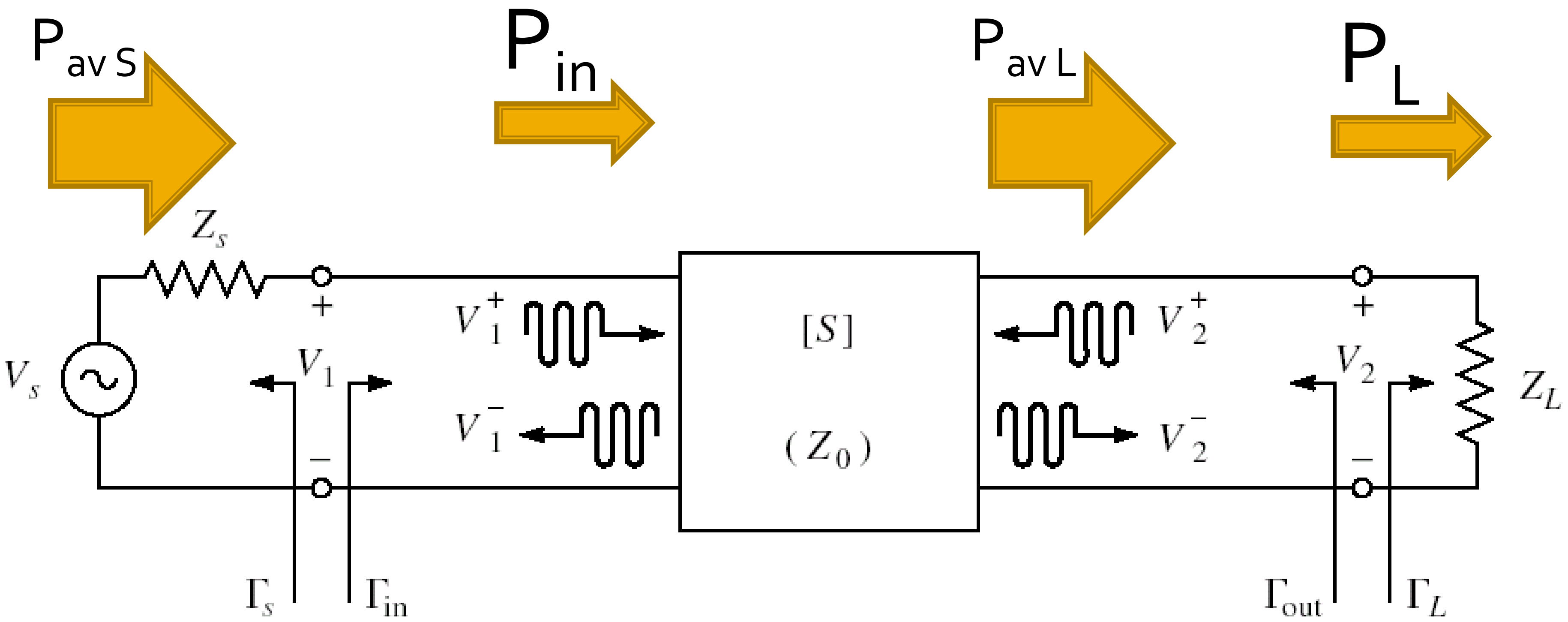
Amplifier as two-port



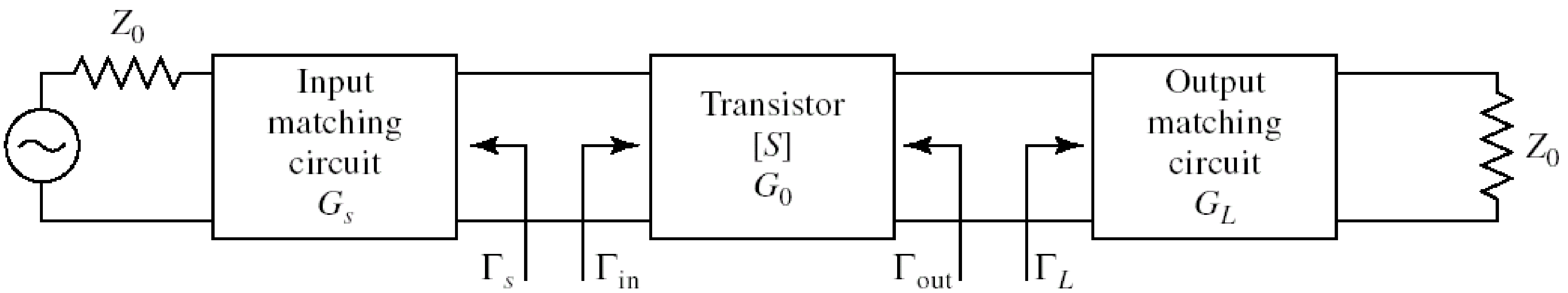
- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Power / Matching

- Two ports in which matching influences the power transfer



Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

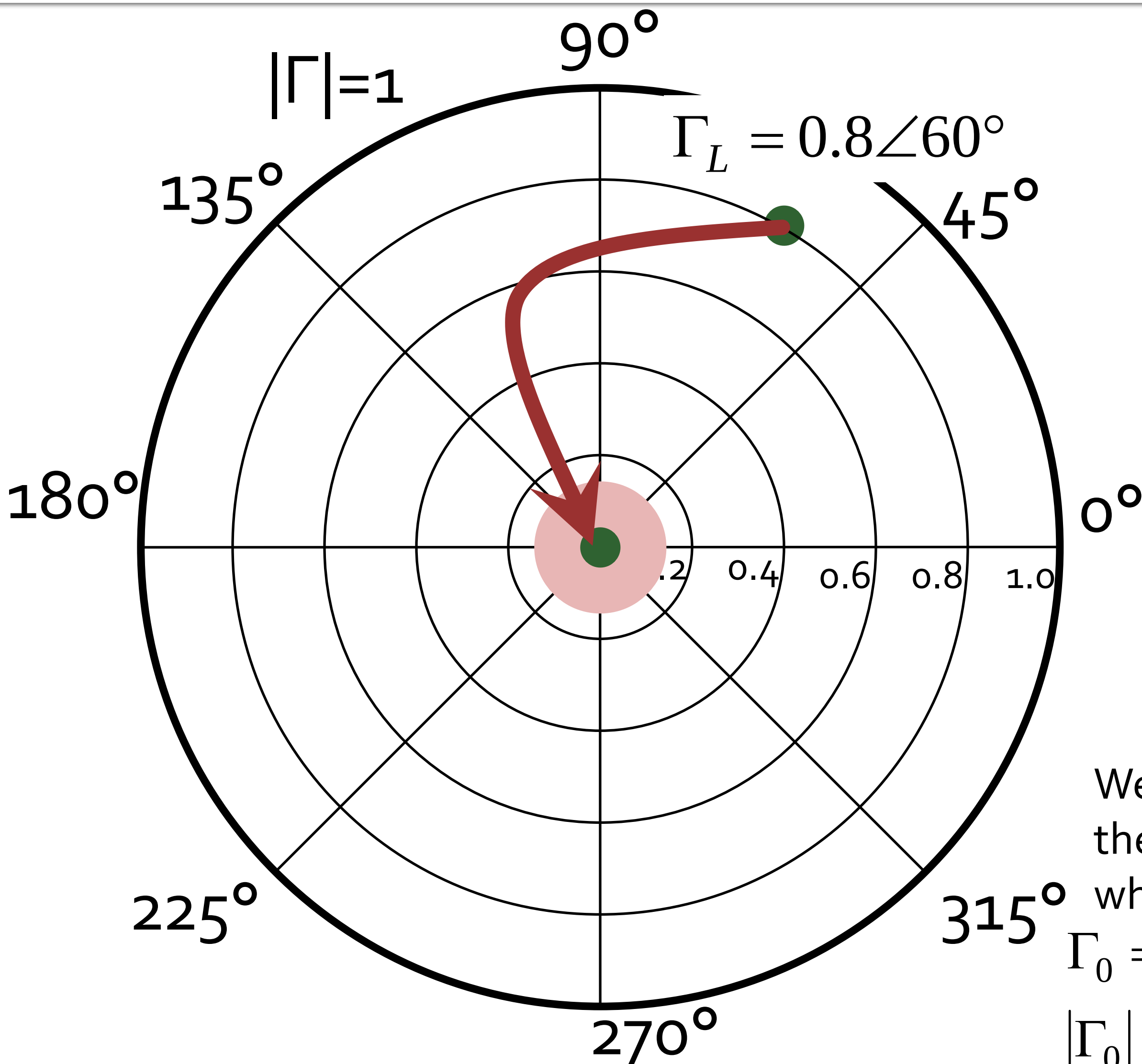
$$\Gamma_{in} = \Gamma_S^* \qquad \Gamma_{out} = \Gamma_L^*$$

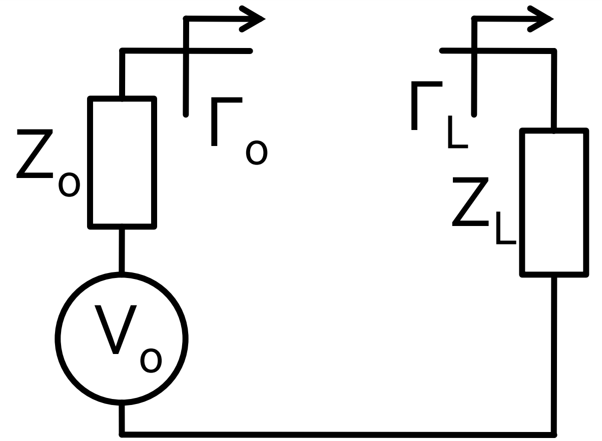
- For lossless matching sections

$$G_{T\max} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right) \cdot \left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S \cdot \Gamma_{in}|^2 \cdot |1-S_{22} \cdot \Gamma_L|^2} \qquad G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22} \cdot \Gamma_L|^2}$$

- For the general case of the bilateral transistor ($S12 \neq 0$) Γ_{in} and Γ_{out} depend on each other so the input and output sections must be matched simultaneously

The Smith Chart, matching, $Z_L \neq Z_o$





Matching Z_L load to Z_o source.

We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

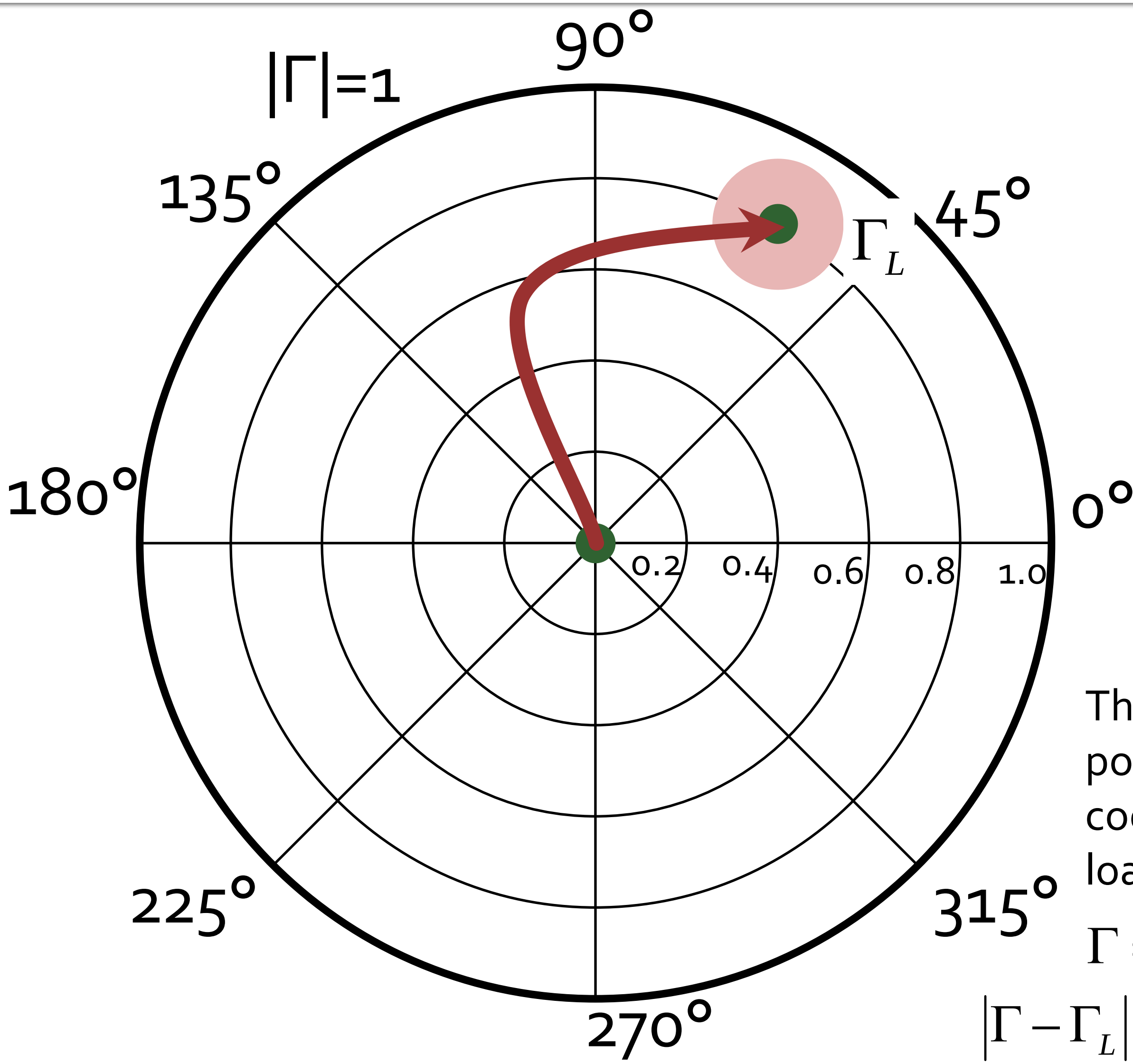
$$\Gamma_L = 0.8\angle 60^\circ$$

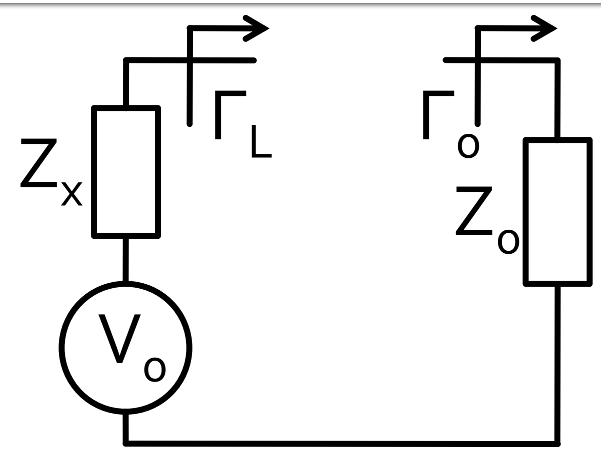
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ “good enough” match

The Smith Chart, matching, $Z_L = Z_o$





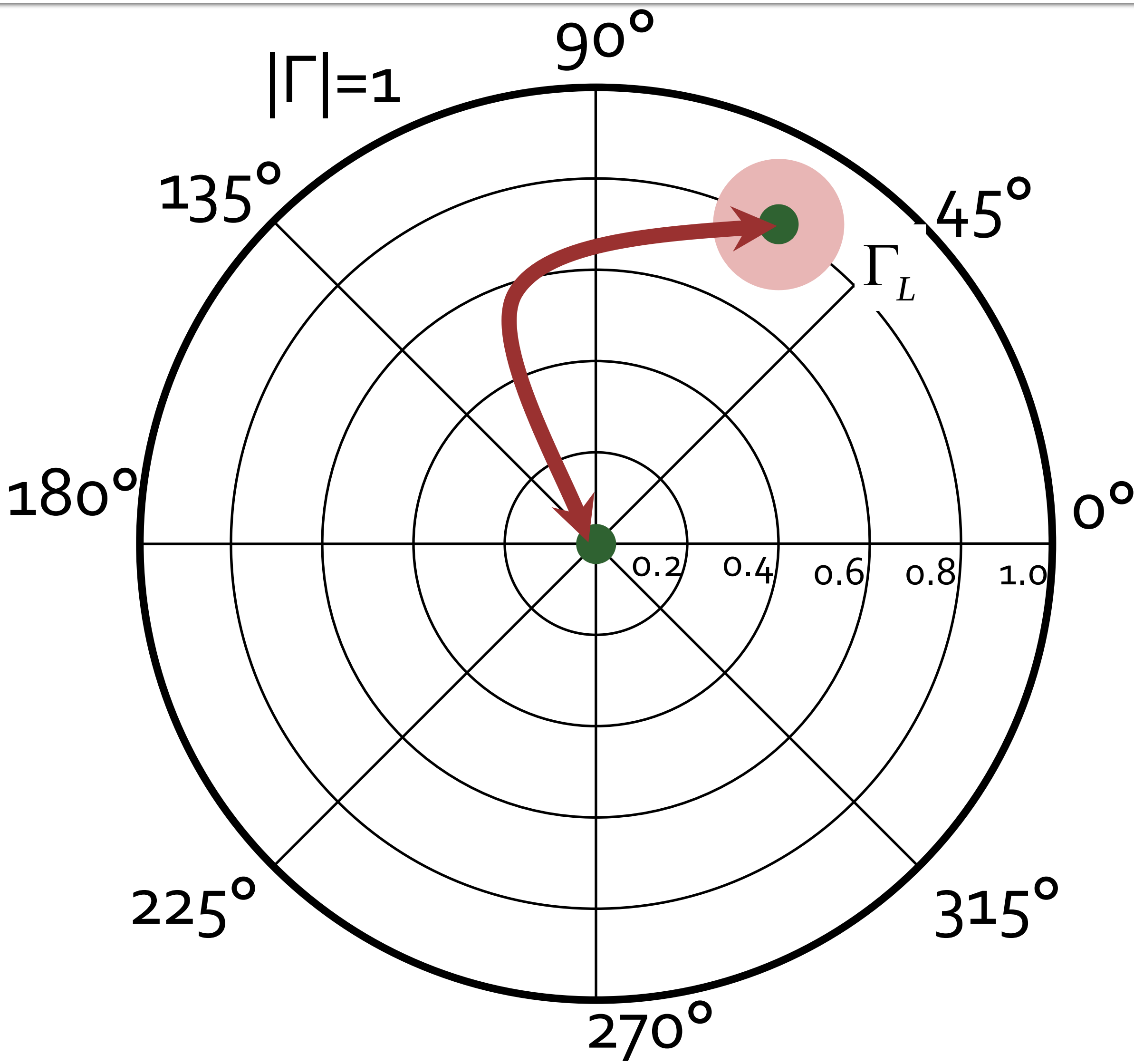
The source (eg. the transistor) having Z_X needs to see a certain reflection coefficient Γ_L towards the load Z_o

The matching circuit must move the point denoting the reflection coefficient in the area where for a Z_o load (Γ_o=0) we see towards it:

$\Gamma = \Gamma_L$ perfect match

$|\Gamma - \Gamma_L| \le \Gamma_m$ "good enough" match

The Smith Chart, matching , $Z_L \neq Z_o$, $Z_L = Z_o$



- The matching sections needed to move
  - Γ_L in Γ_o
  - Γ_o in Γ_L
- are **identical**. They differ only by the **order** in which the elements are introduced into the matching circuit
- As a result, we can use in match design the same:
  - **methods**
  - **formulae**

Simultaneous matching

$$\Gamma_{in} = \Gamma_S^*$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_S^* = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \Gamma_L^*$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

- We find Γ_S

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* \cdot S_{21}^*}{1/\Gamma_L^* - S_{22}^*}$$

$$\Gamma_L^* = \frac{S_{22} - \Delta \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_S \cdot \left(1 - |S_{22}|^2\right) + \Gamma_S^2 \cdot \left(\Delta \cdot S_{22}^* - S_{11}\right) = \Gamma_S \cdot \left(\Delta \cdot S_{11}^* \cdot S_{22}^* - |S_{22}|^2 - \Delta \cdot S_{12}^* \cdot S_{21}^*\right) + S_{11}^* \cdot \left(1 - |S_{22}|^2\right) + S_{12}^* \cdot S_{21}^* \cdot S_{22}$$

Simultaneous matching

$$\Delta \cdot \left(S_{11}^* \cdot S_{22}^* - S_{12}^* \cdot S_{21}^*\right) = |\Delta|^2$$

$$\Gamma_S^2 \cdot \underbrace{\left(S_{11} - \Delta \cdot S_{22}^*\right)}_{C} + \Gamma_S \cdot \underbrace{\left(|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1\right)}_{-B} + \underbrace{\left(S_{11}^* - \Delta^* \cdot S_{22}\right)}_{C^*} = 0$$

- A quadratic equation

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

- Similarly

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

- With variables defined as:

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \quad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

Simultaneous matching

- Simultaneous matching is possible if:

$$B_1^2 - 4 \cdot |C_1|^2 > 0 \qquad B_2^2 - 4 \cdot |C_2|^2 > 0$$

$$\Delta \cdot \left(S_{11}^* \cdot S_{22}^* - S_{12}^* \cdot S_{21}^* \right) = |\Delta|^2$$

$$|C_1|^2 = \left| S_{11} - \Delta \cdot S_{22}^* \right|^2 = |S_{12}|^2 \cdot |S_{21}|^2 + \left(1 - |S_{22}|^2\right) \cdot \left(|S_{11}|^2 - |\Delta|^2\right)$$

$$B_1^2 - 4 \cdot |C_1|^2 = \left(1 + |S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$- 2 \cdot \left(1 + |S_{11}|^2\right) \cdot \left(|S_{22}|^2 + |\Delta|^2\right) - 4 \cdot |S_{12} \cdot S_{21}|^2 - 4 \cdot \left(1 - |S_{22}|^2\right) \cdot \left(|S_{22}|^2 - |\Delta|^2\right)$$

$$B_1^2 - 4 \cdot |C_1|^2 = \left(1 + |S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$- 4 \cdot |S_{11}|^2 - 4 \cdot |S_{22}|^2 \cdot |\Delta|^2 - 2 \cdot \left(1 - |S_{11}|^2\right) \cdot \left(|S_{22}|^2 - |\Delta|^2\right) - 4 \cdot |S_{12} \cdot S_{21}|^2$$

Simultaneous matching

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 - \\ -4\cdot|S_{11}|^2 - 4\cdot|S_{22}|^2\cdot|\Delta|^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2\right)^2 + \left(|S_{22}|^2 - |\Delta|^2\right)^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(K\cdot 2\cdot|S_{12}\cdot S_{21}|\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2 - 1\right)$$

- Similarly

$$B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2 - 1\right)$$

Simultaneous matching

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

- Solutions must ensure stability:

$$|\Gamma_S| < 1 \qquad |\Gamma_L| < 1$$

$$|\Delta| = |S_{11}\cdot S_{22} - S_{12}\cdot S_{21}| < 1 \qquad \begin{cases} B_1 > 0 \\ B_2 > 0 \end{cases}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\cdot|S_{12}\cdot S_{21}|} > 1 \qquad \begin{cases} B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2 - 1\right) > 0 \\ B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2 - 1\right) > 0 \end{cases}$$

Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and $|\Gamma|<1$ solutions are those with "–" sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta\cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta\cdot S_{11}^* \end{cases}$$

Simultaneous matching

- In the case of the simultaneous matching the amplifier achieves the maximum transducer power gain for the bilateral transistor

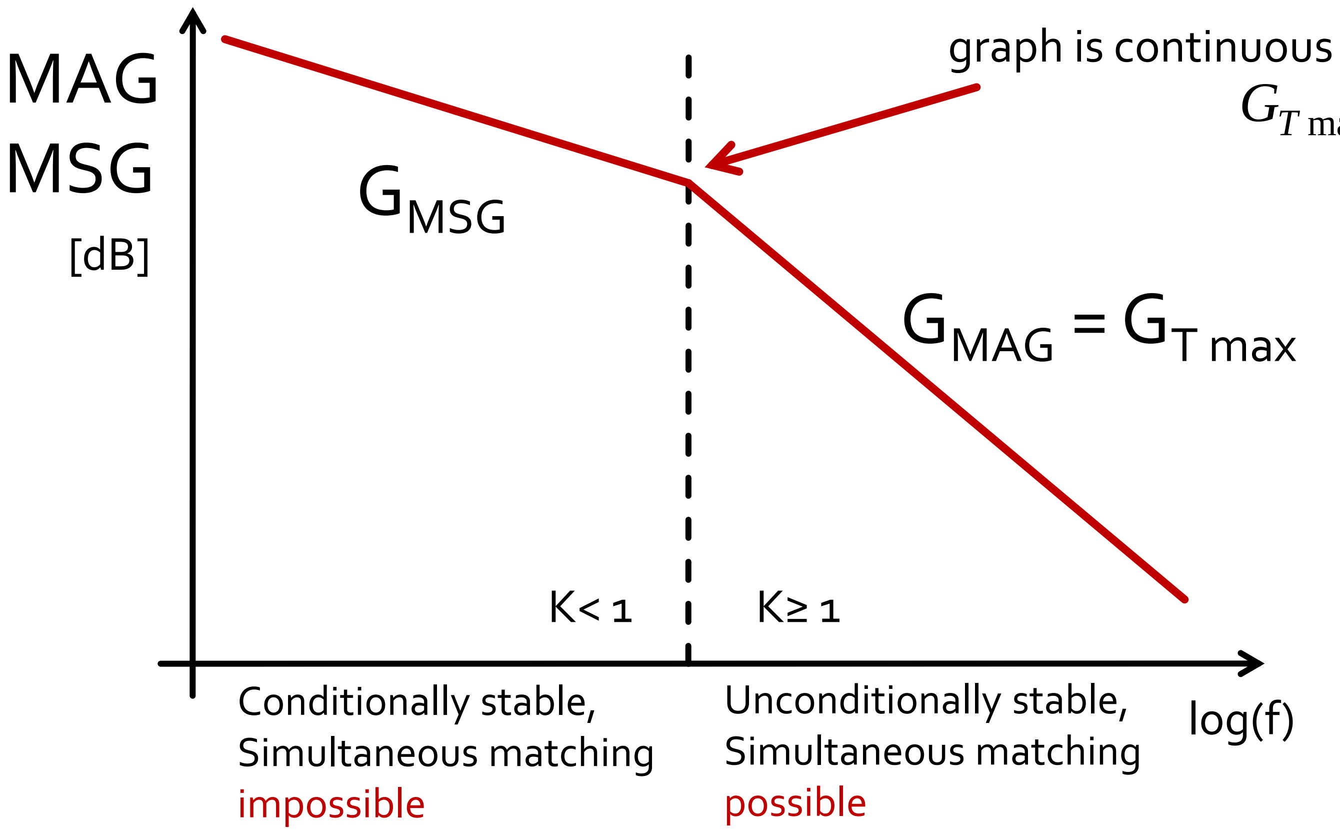
$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right)$$

- If the device is **not unconditionaly stable** at a certain frequency we can use MSG (Maximum Stable Gain) as an indicator of the capability to obtain a power gain in stable conditions

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

Maximum Available Gain

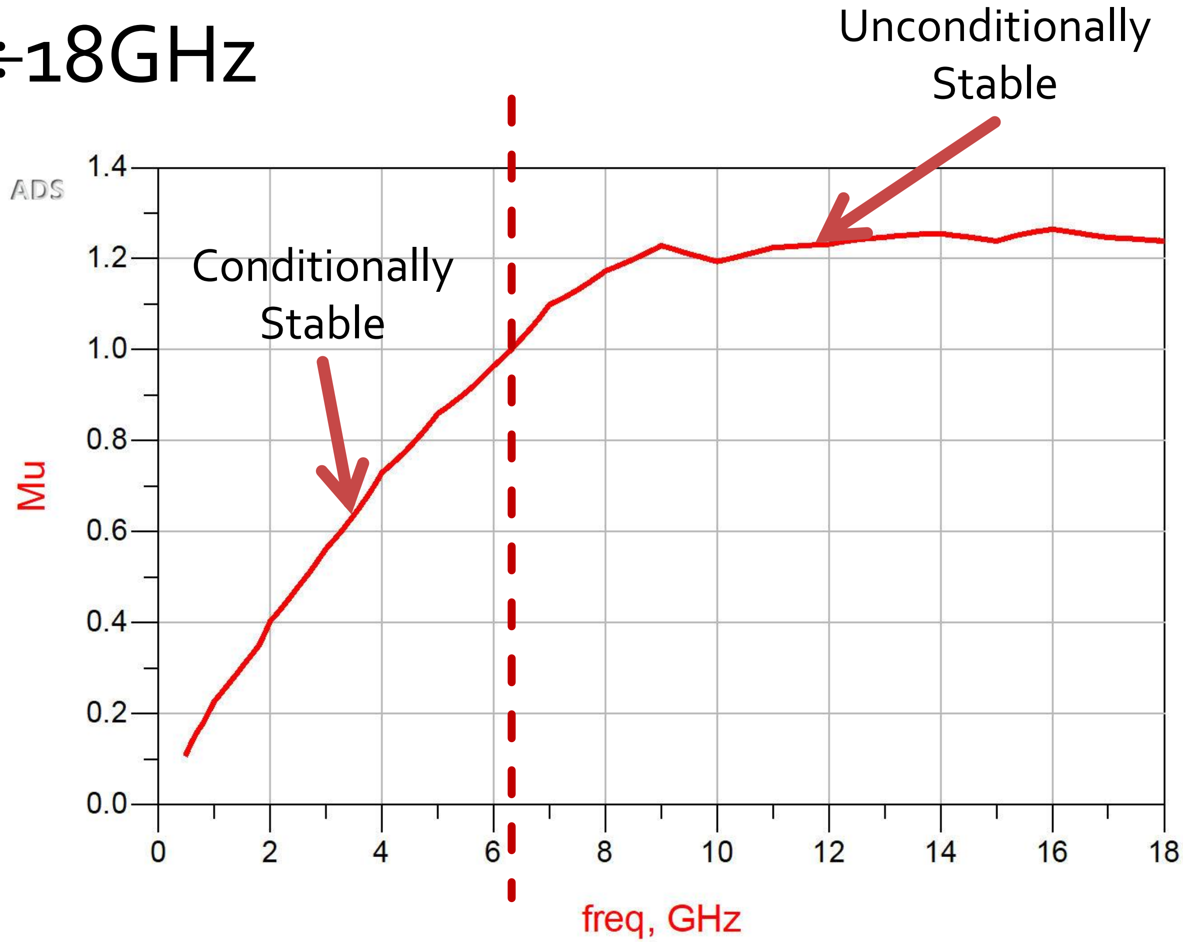
- Indicator across full frequency range of the capability to obtain a power gain



$$G_{T\max}\Big|_{K=1} = \frac{|S_{21}|}{|S_{12}|} = G_{MSG}$$

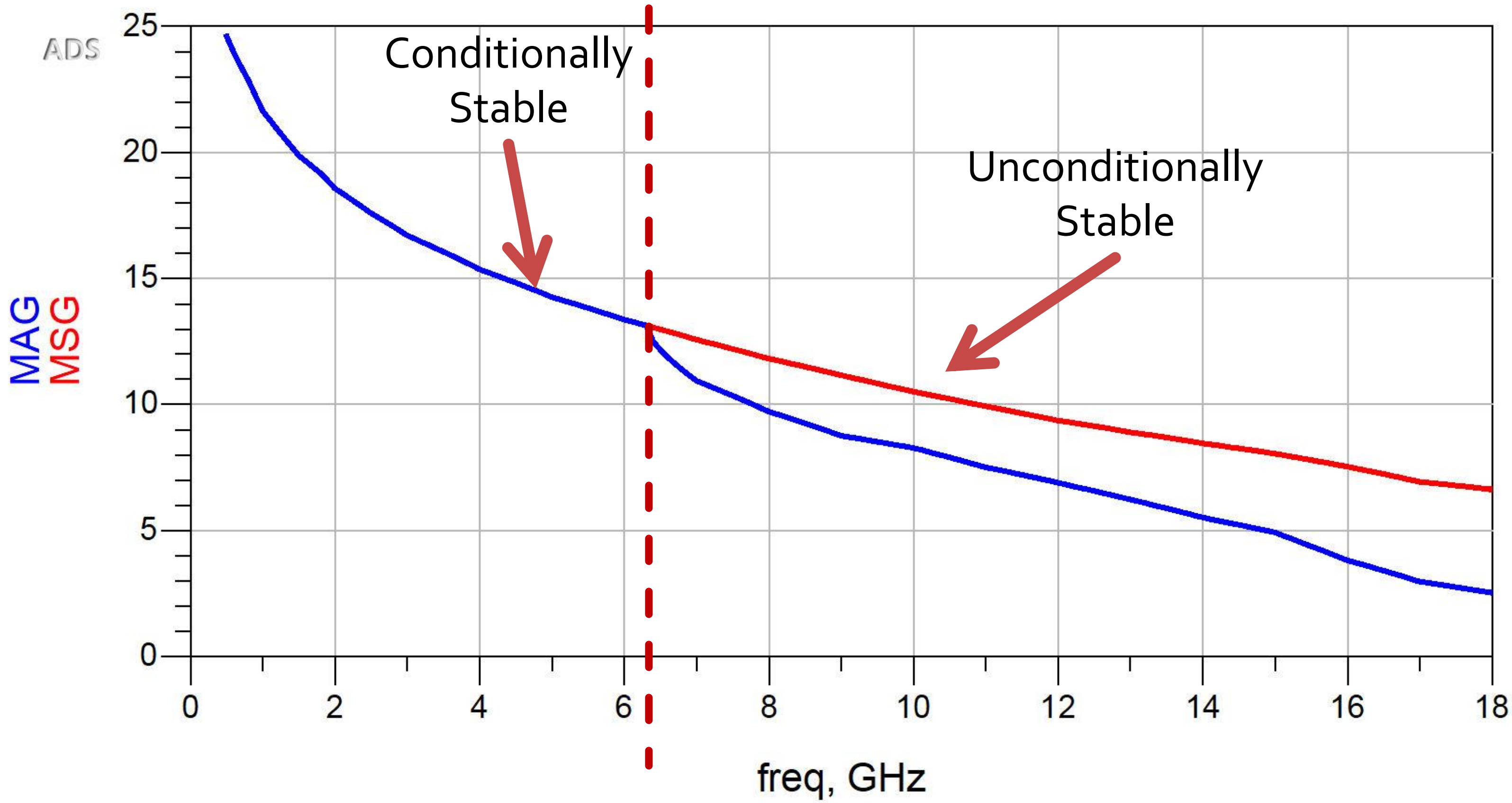
Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



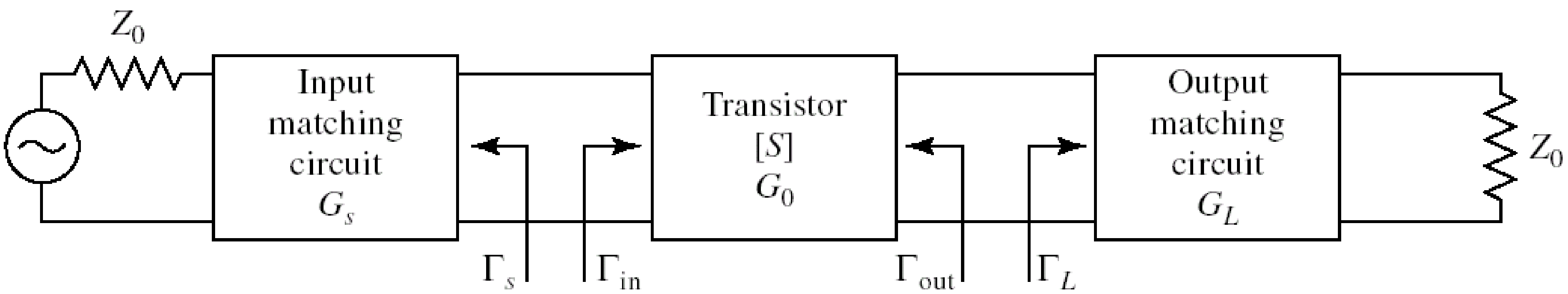
MAG/MSG

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Simultaneous matching, unilateral transistor

- In the case of **unilateral** amplifier/transistor (S12 = 0) simultaneous matching implies:



$$\Gamma_{in} = S_{11} \qquad \Gamma_{out} = S_{22}$$

$$\Gamma_S = S_{11}^* \qquad \Gamma_L = S_{22}^*$$

$$G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2} \qquad G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
  - without stabilization K = **0.886**, MAG = 14.248dB @ 5GHz
  - cannot be used with this bias conditions
- ATF-34143 **at Vds=4V Id=40mA**
  - without stabilization K = **1.031**, MAG = 12.9dB @ 5GHz
  - we use this bias conditions for simultaneous matching

Example

- ATF-34143 **at Vds=4V Id=40mA**.
- @5GHz
  - $S_{11} = 0.64\angle 111°$
  - $S_{12} = 0.117\angle -27°$
  - $S_{21} = 2.923\angle -6°$
  - $S_{22} = 0.21\angle 111°$

Computations

- Complex S Parameters
  - S11 = -0.229+0.597·j
  - S12 = 0.104-0.053·j
  - S21 = 2.907-0.306·j
  - S22 = -0.075+0.196·j

$$\begin{cases} S_{11} = 0.64\angle 111^\circ \\ S_{11} = 0.64 \cdot \cos 111^\circ + j \cdot 0.64 \cdot \sin 111^\circ \end{cases}$$

$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right) = 19.497 = 12.9\,\text{dB}$$

$$G_{TU\max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} = 15.139 = 11.8\,\text{dB}$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = ? \\ C_1 = ? \end{cases}$$

$$\begin{cases} B_2 = ? \\ C_2 = ? \end{cases}$$

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = ?$$

$$\Gamma_L = ?$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = 1.207 \\ C_1 = -0.277 + j \cdot 0.529 \end{cases}$$

$$\begin{cases} B_2 = 0.476 \\ C_2 = -0.222 - j \cdot 0.013 \end{cases}$$

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = -0.403 - j \cdot 0.768$$

$$|\Gamma_S| = 0.867 < 1$$

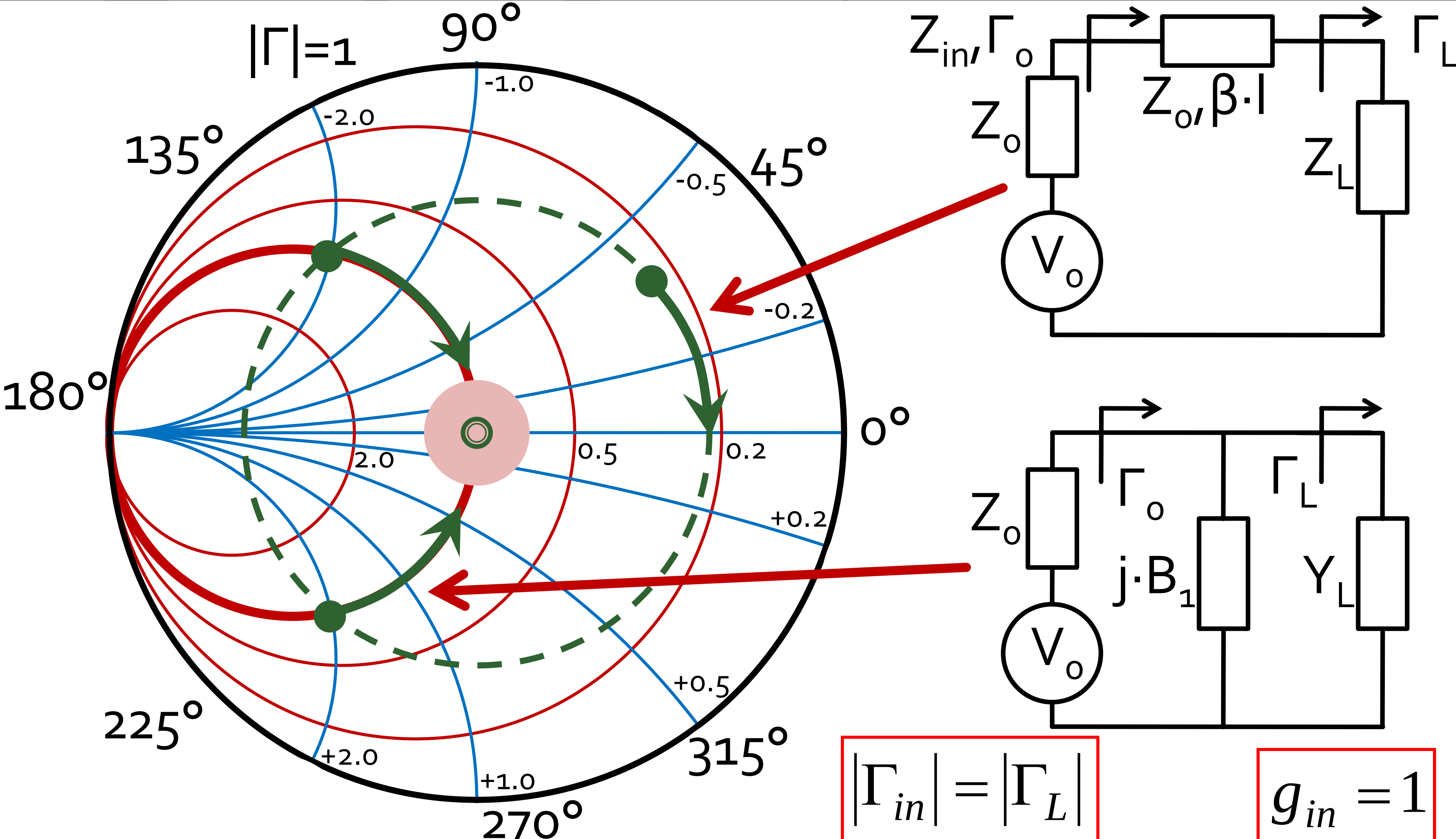
$$\Gamma_S = 0.867 \angle -117.7°$$

$$\Gamma_L = -0.685 + j \cdot 0.04$$

$$|\Gamma_L| = 0.686 < 1$$

$$\Gamma_L = 0.686 \angle 176.7°$$

Shunt stub matching, L8



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution (Γ_S)

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.867 \angle -117.7°$$

$$|\Gamma_S| = 0.867; \quad \varphi = -117.7° \qquad \cos(\varphi + 2\theta) = -0.867 \Rightarrow \quad (\varphi + 2\theta) = \pm 150.1°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(-117.7° + 2\theta) = +150.1° \qquad \theta = 133.9° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -3.477$$

$$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -74° (+180°) \rightarrow \theta_{sp} = 106°$$

  - **"-" solution**

$$(-117.7° + 2\theta) = -150.1° \qquad \theta = -16.2° (+180°) \rightarrow \theta = 163.8°$$

$$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +3.477 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 74°$$

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686\angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**
  - **"-" solution**

Analytical solution (Γ_L)

$$\cos(\varphi+2\theta)=-|\Gamma_L|$$

$$\theta_{sp}=\beta\cdot l=\tan^{-1}\frac{\mp 2\cdot|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}}$$

$$\Gamma_L=0.686\angle 176.7°$$

$$|\Gamma_L|=0.686;\quad \varphi=176.7° \qquad \cos(\varphi+2\theta)=-0.686\Rightarrow\quad(\varphi+2\theta)=\pm 133.3°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(176.7°+2\theta)=+133.3° \qquad \theta=-21.7°(+180°)\rightarrow\theta=158.3°$$

$$\theta_{sp}=\tan^{-1}(\operatorname{Im}y_L)=-62.1°(+180°)\rightarrow\theta_{sp}=117.9° \qquad \operatorname{Im}y_L=\frac{-2\cdot|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}}=-1.885$$

  - **"-" solution**

$$(176.7°+2\theta)=-133.3° \qquad \theta=-155°(+180°)\rightarrow\theta=25°$$

$$\operatorname{Im}y_L=\frac{+2\cdot|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}}=+1.885 \qquad \theta_{sp}=\tan^{-1}(\operatorname{Im}y_L)=62.1°$$

Complete analytical solution

- We choose **one** of the two possible solutions for the input matching

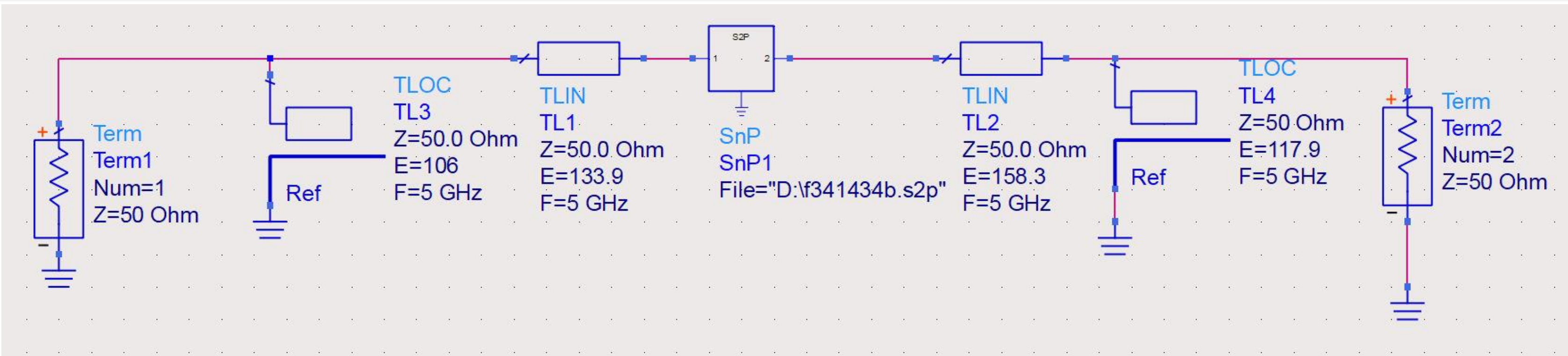
$$(\varphi + 2\theta) = \begin{cases} +150.1^\circ \\ -150.1^\circ \end{cases} \quad \theta = \begin{cases} 133.9^\circ \\ 163.8^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -3.477 \\ +3.477 \end{cases} \quad \theta_{sp} = \begin{cases} -74^\circ + 180^\circ = 106^\circ \\ +74^\circ \end{cases}$$

- Similarly for the output matching

$$(\varphi + 2\theta) = \begin{cases} +133.3^\circ \\ -133.3^\circ \end{cases} \quad \theta = \begin{cases} 158.3^\circ \\ 25.0^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -1.885 \\ +1.885 \end{cases} \quad \theta_{sp} = \begin{cases} 117.9^\circ \\ 62.1^\circ \end{cases}$$

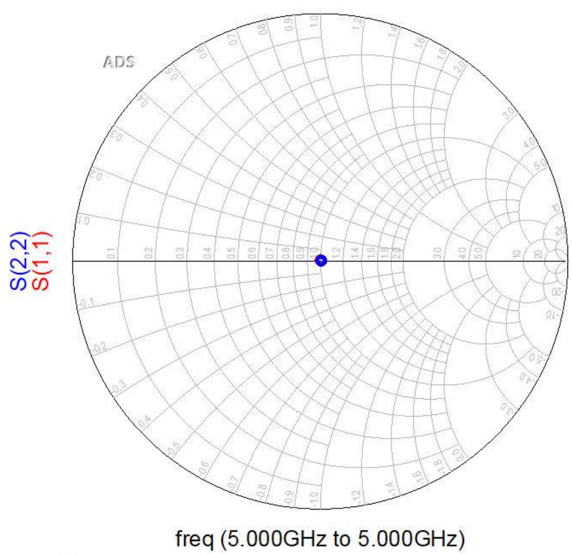
- In total there are **4** possible solutions input/output

ADS

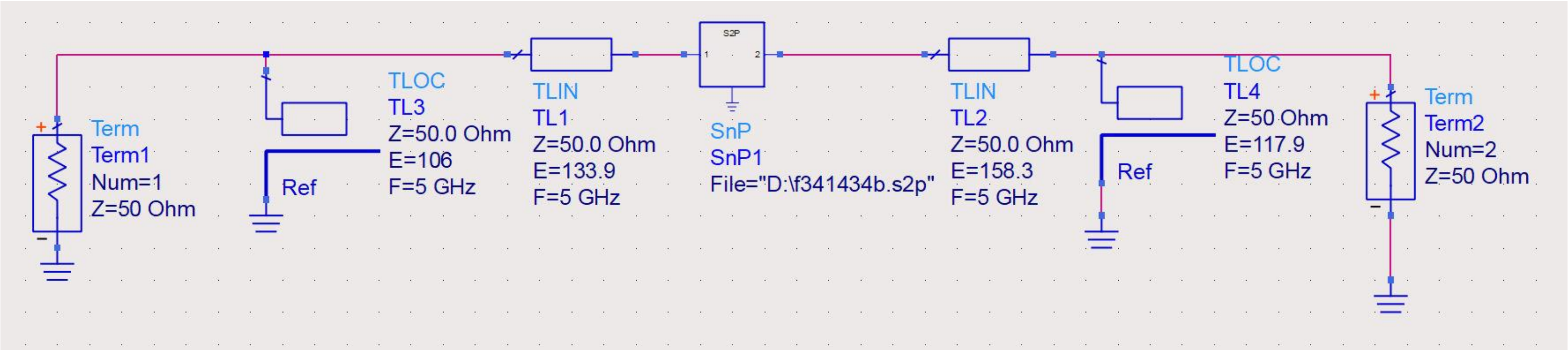


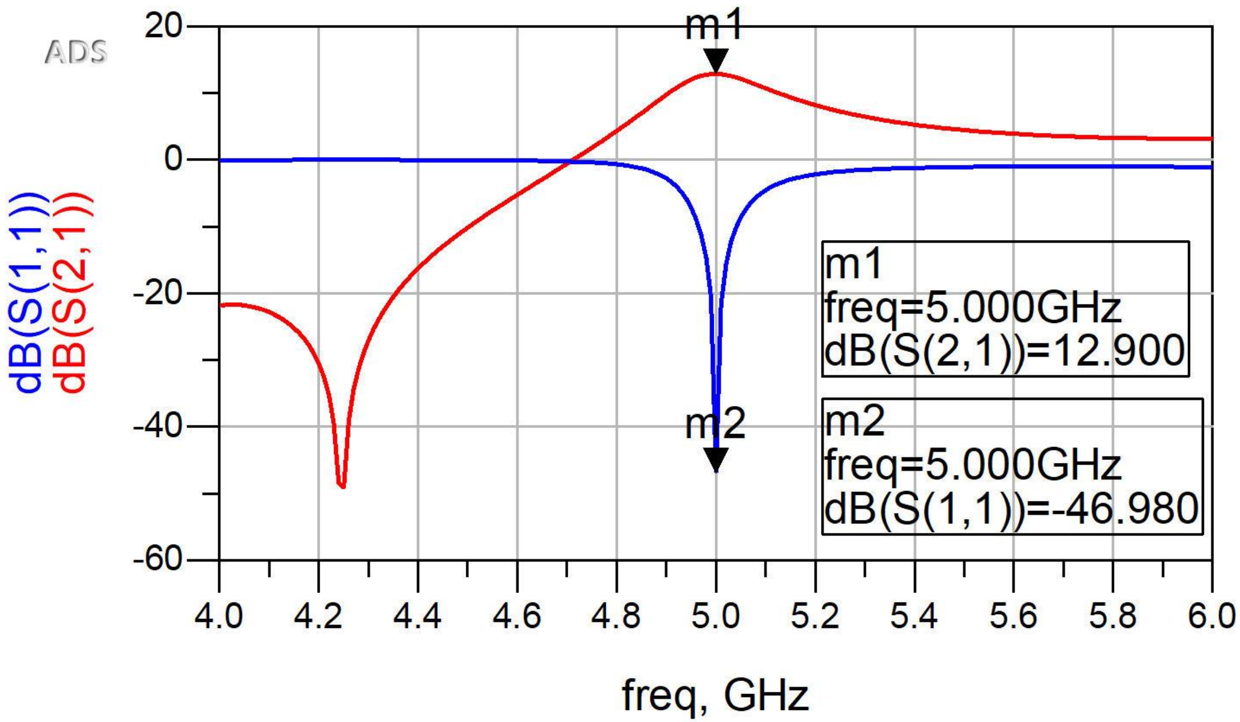
Eqn GT=10*log(mag(S(2,1))**2)

freq	S(2,1)	GT	S(1,1)	S(2,2)
5.000 GHz	4.415 / 157.353	12.900	0.004 / 86.088	0.004 / 37.766



ADS

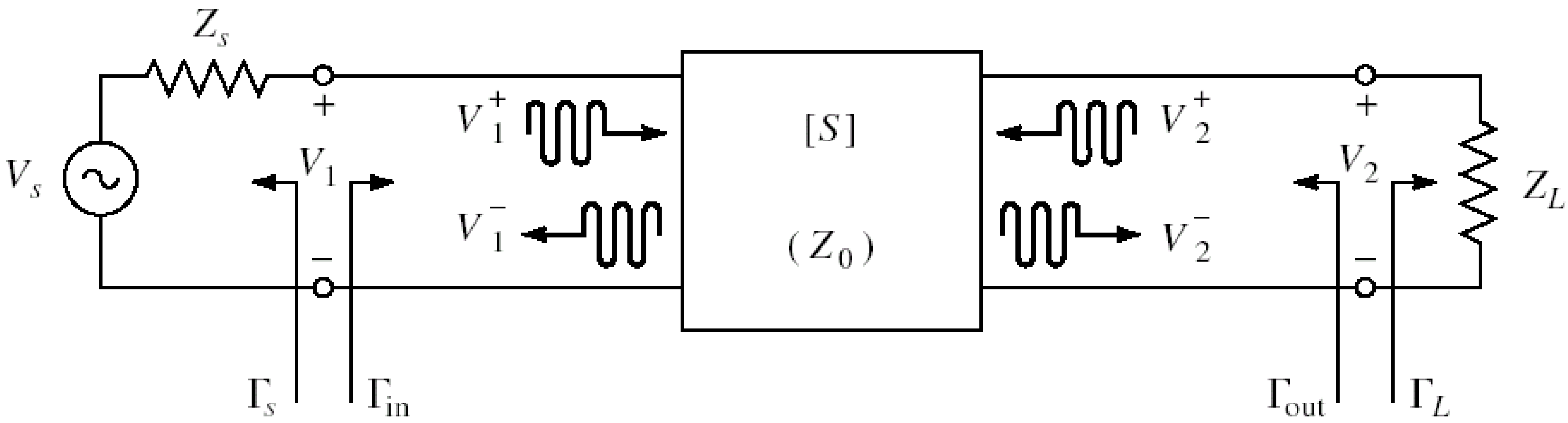

Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=106
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=133.9
F=5 GHz
S2P
SnP
SnP1
File="D:\f341434b.s2p"
TLIN
TL2
Z=50.0 Ohm
E=158.3
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=117.9
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm


ADS
dB(S(1,1))
dB(S(2,1))
m1
m2
m1
freq=5.000GHz
dB(S(2,1))=12.900
m2
freq=5.000GHz
dB(S(1,1))=-46.980
20
0
-20
-40
-60
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz

Microwave Amplifiers

Design for Specified Gain

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

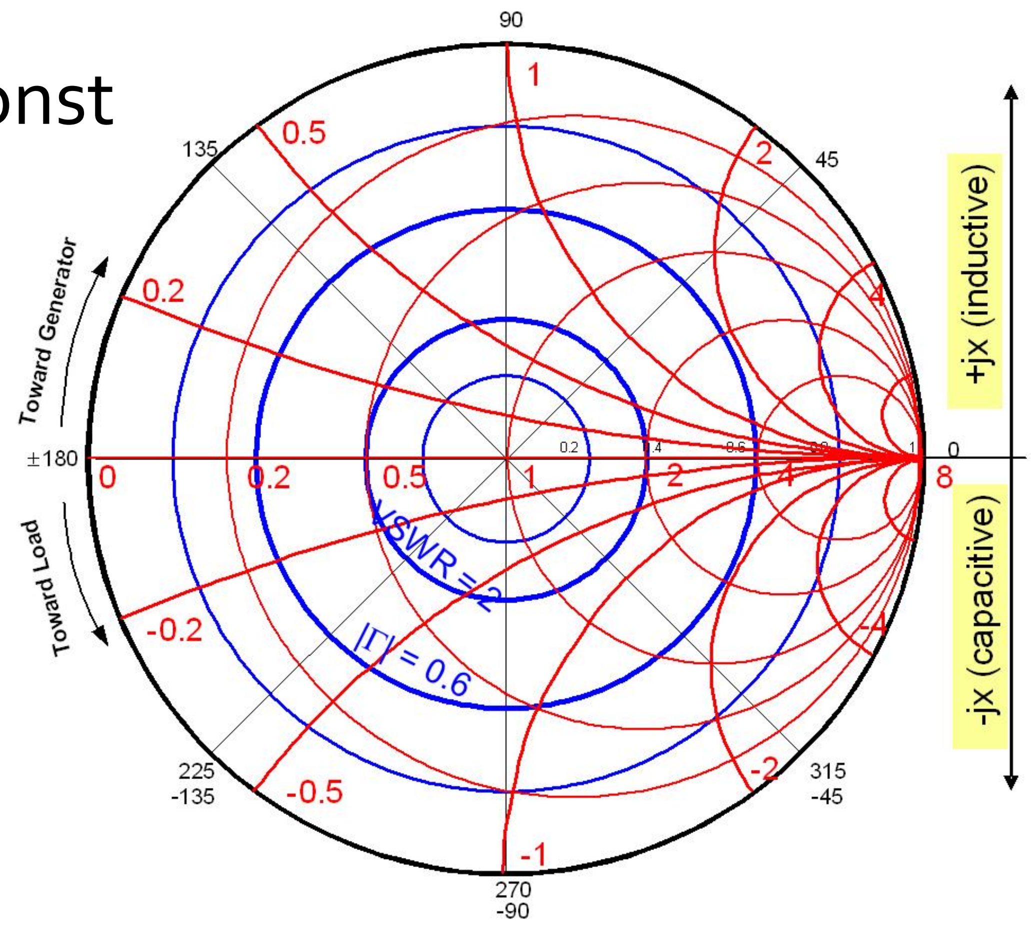
Design for Specified Gain

- In many cases we need an approach other than "brute force" when we prefer to design for **less than the maximum obtainable gain**, in order to:
  - improve noise behavior (Lab 3 + Lect. 10 next)
  - improve stability
  - improve VSWR
  - control performance at multiple frequencies
  - improve amplifier's bandwidth

Constant VSWR circles

- Certain applications may require a certain ratio between maximum / minimum line voltage
  - VSWR = const → Γ = const

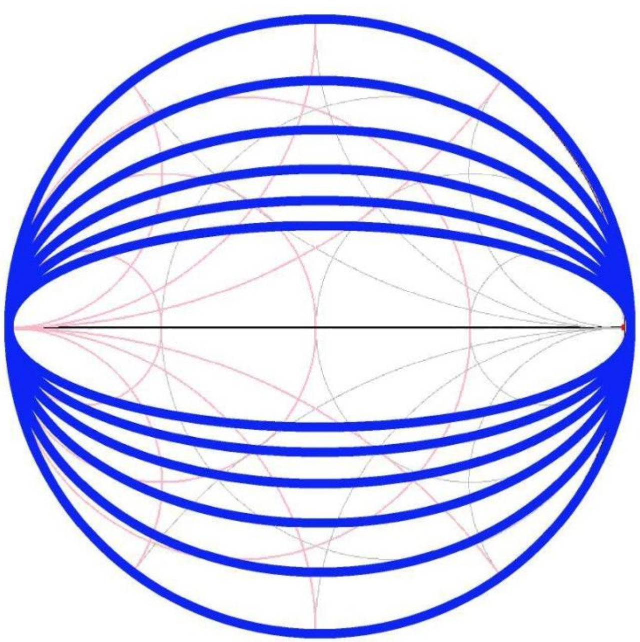
$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

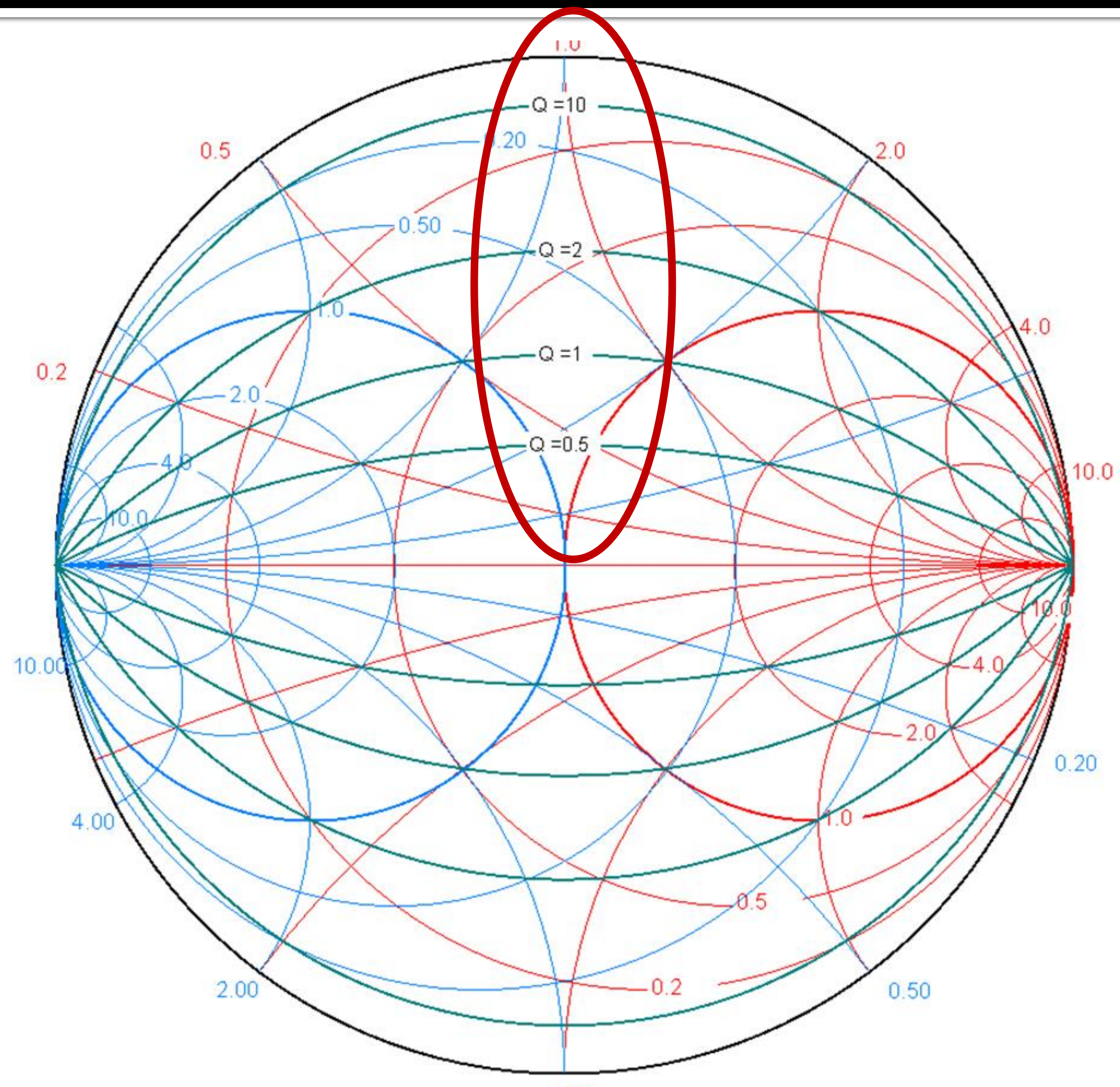


Constant Q circles

- Quality factor Q

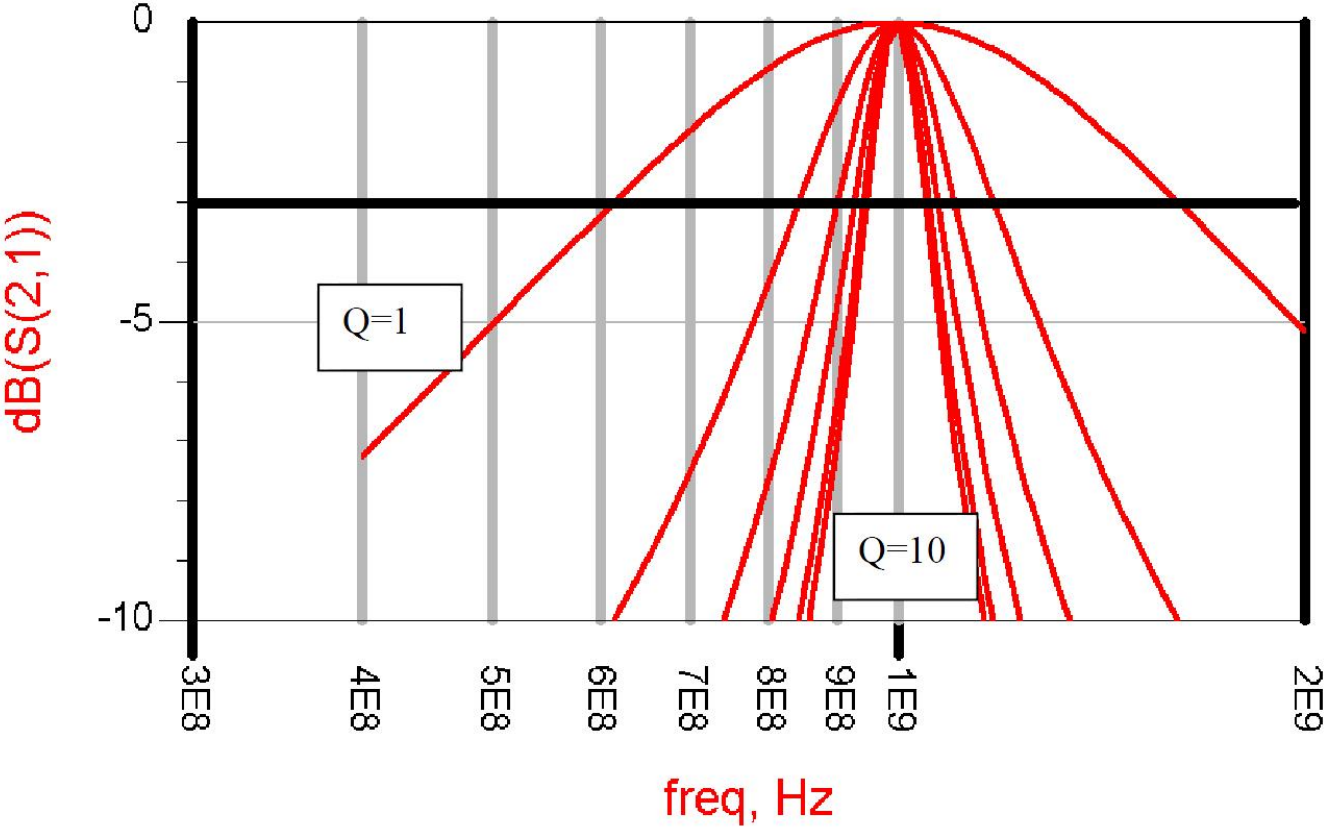
$$Q = \frac{X}{R} = \frac{G}{B} = const$$





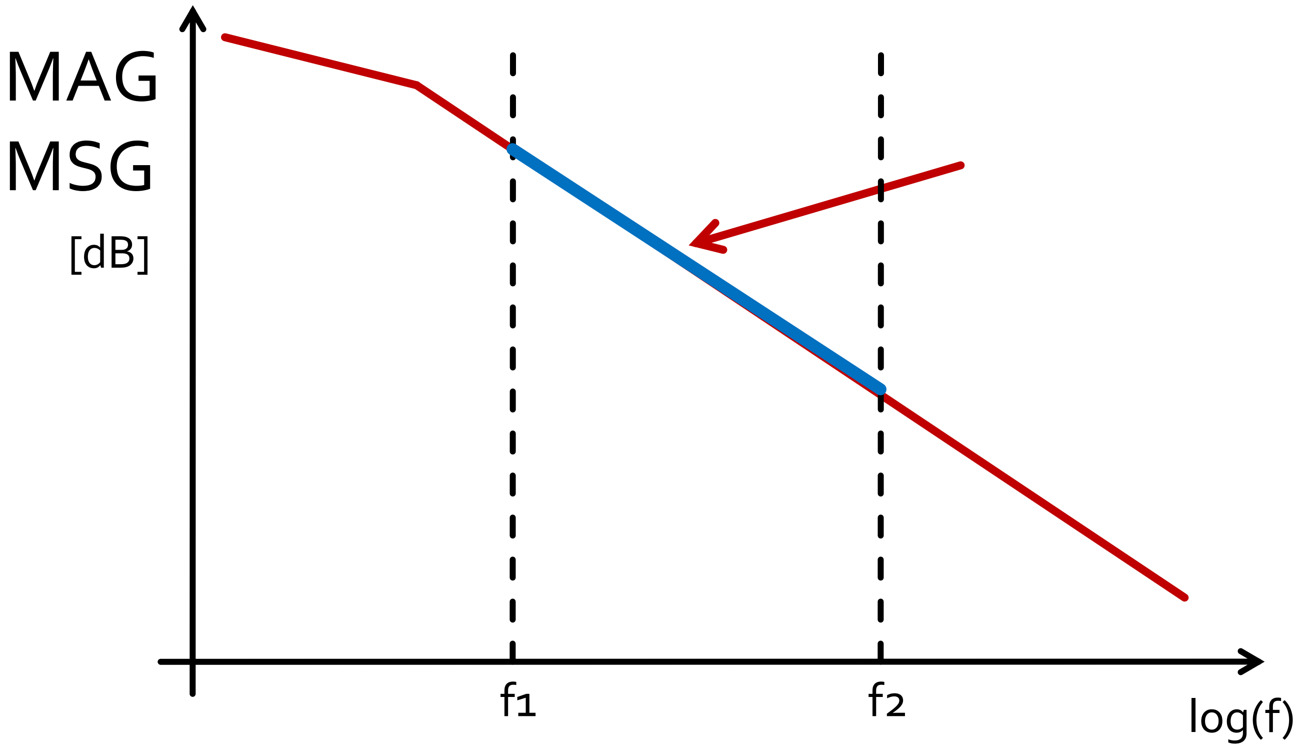
Quality factor - bandwidth

- High quality factor is equivalent with narrow bandwidth



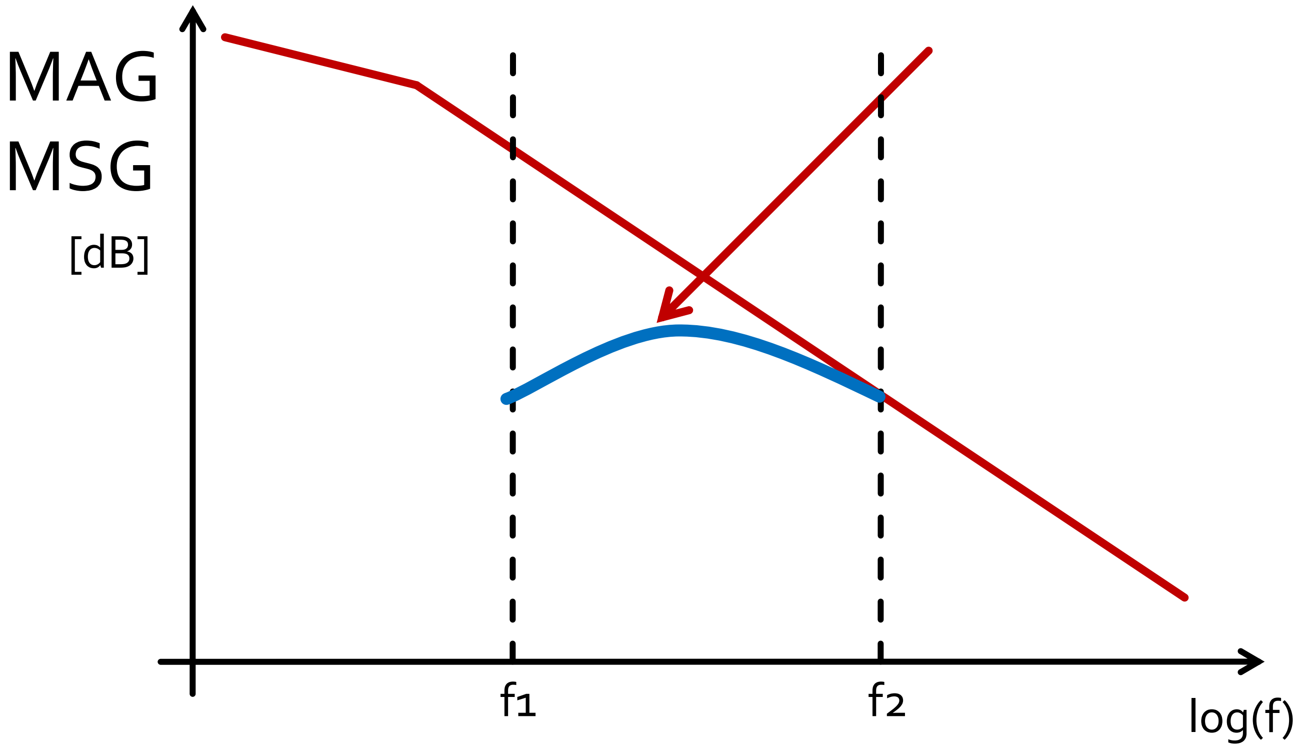
Wide bandwidth amplifier

- Design for maximum gain at two different frequencies creates an frequency unbalanced amplifier



Wide bandwidth amplifier

- Design for maximum gain at highest frequency
- Controlled mismatch at lower frequency
  - eventually at more frequencies inside the bandwidth



Design for Specified Gain

- Assumes the amplifier device **unilateral**

Input and output can be treated independently

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0$$

$$\Gamma_{in} = S_{11}$$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Unilateral figure of merit

- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \qquad U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)}$$

- We compute U then the maximum and minimum deviation of G_{TU} from G_T
  - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20\cdot\log(1+U) < G_T[dB] - G_{TU}[dB] < -20\cdot\log(1-U)$$

Example

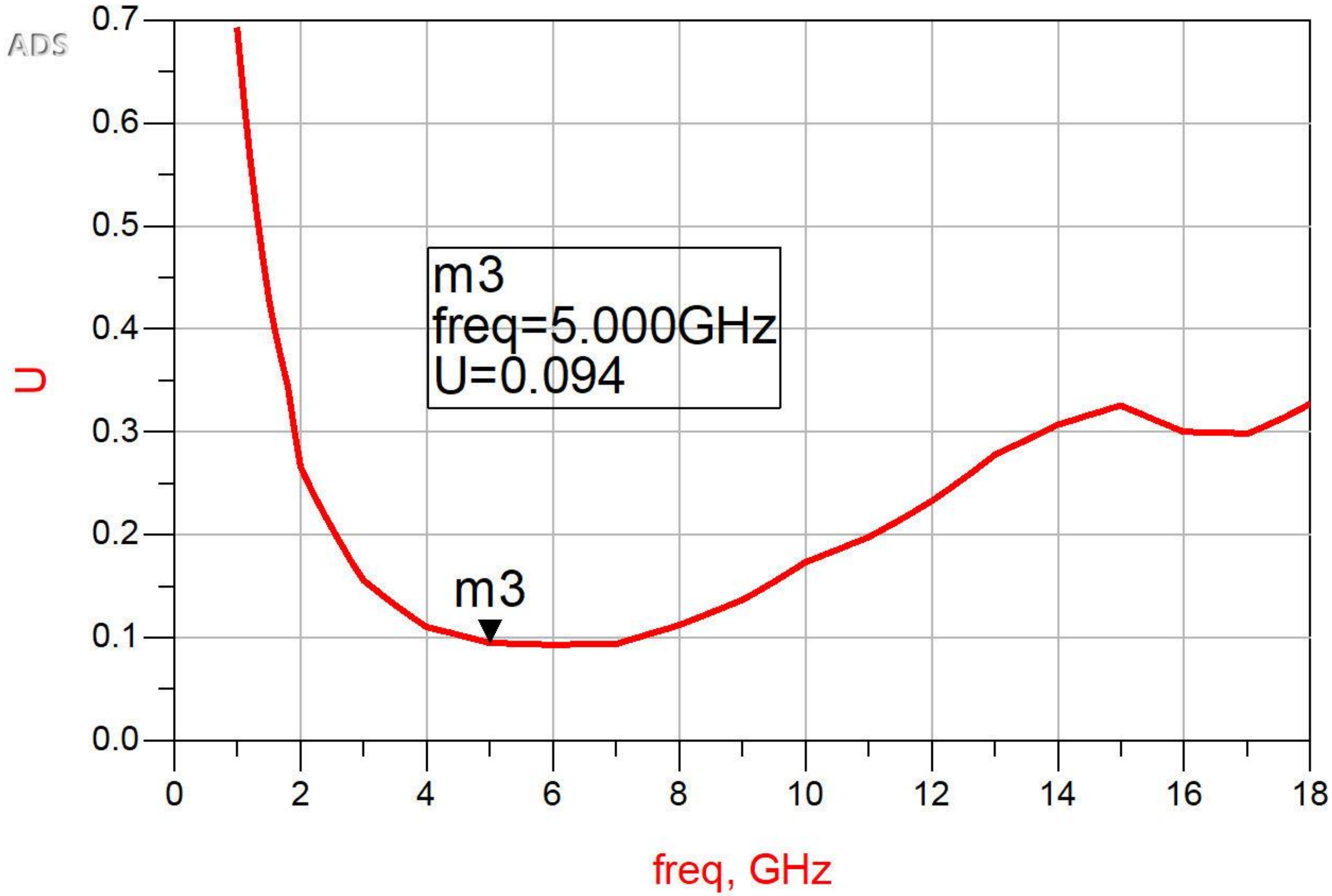
- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

$$U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)} = 0.094$$

$$-0.783\,dB < G_T[dB] - G_{TU}[dB] < 0.861\,dB$$

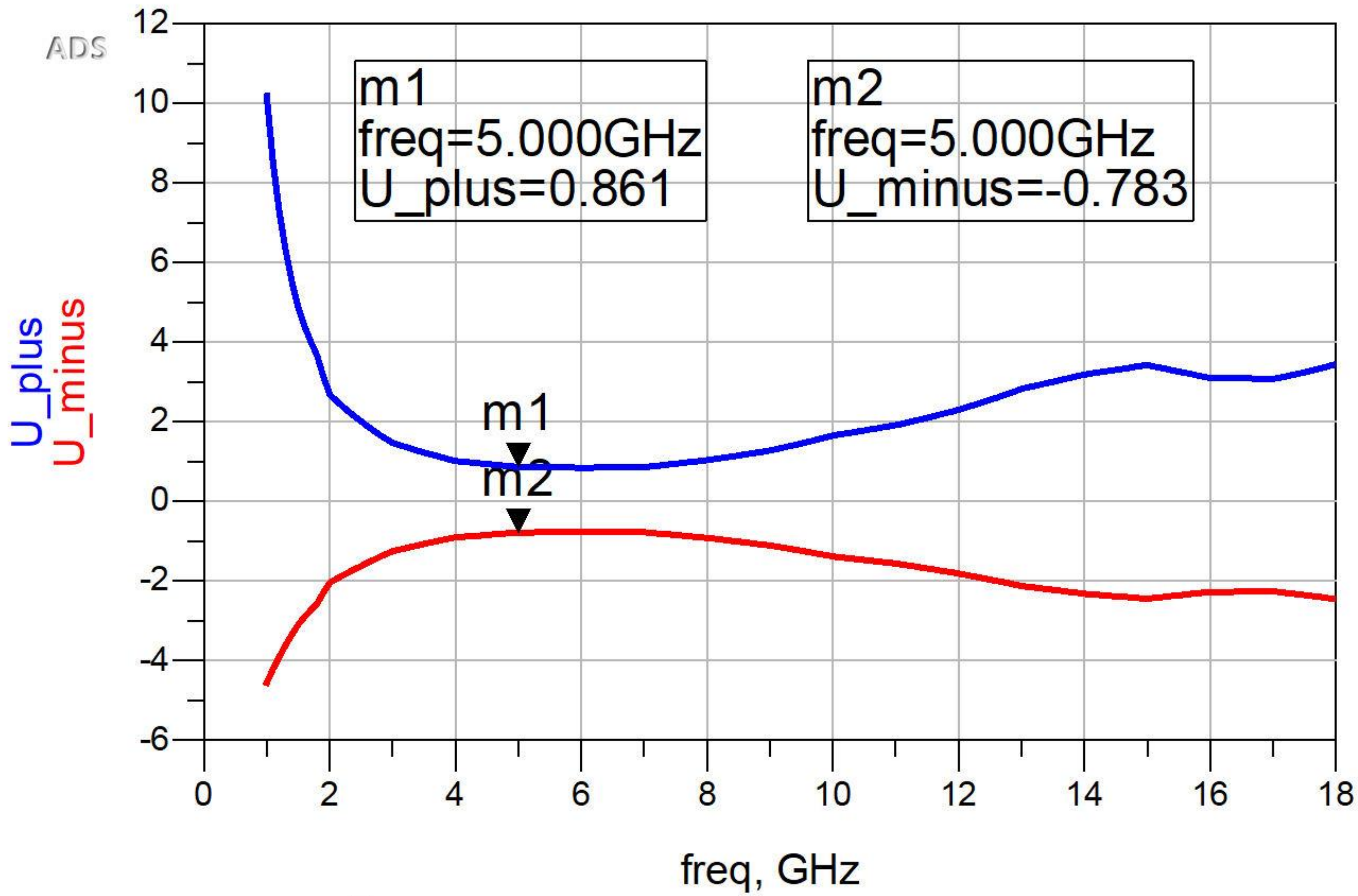
Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz

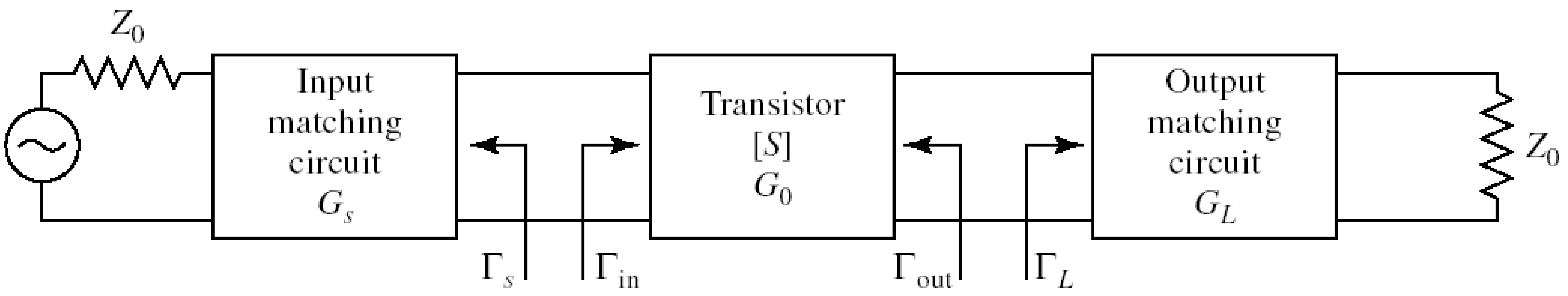


Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz, maximum and minimum deviation [dB]



Design for Specified Gain



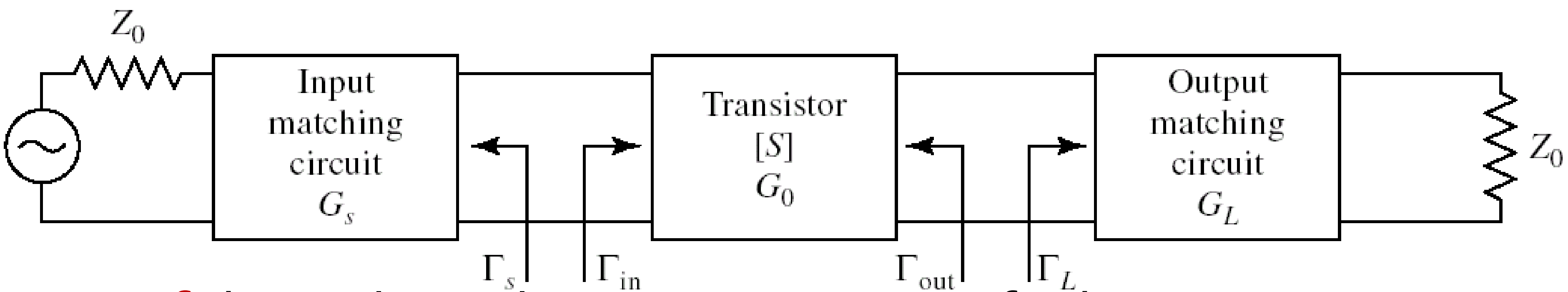
- In the unilateral assumption:

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = G_S(\Gamma_S) \qquad G_L = G_L(\Gamma_L)$$

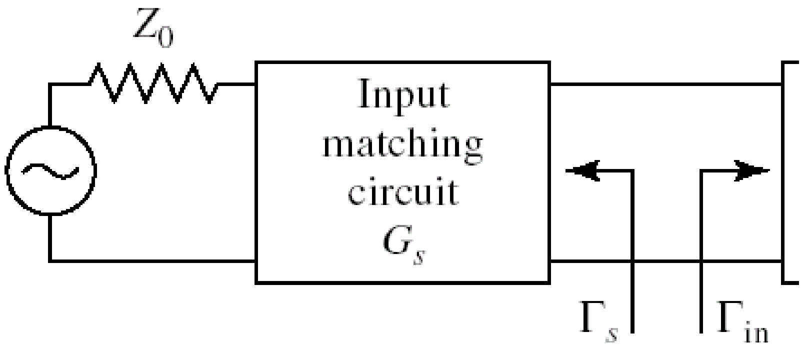
Design for Specified Gain



- **If** the unilateral assumption is justified :
  - power gain added by the input matching circuit **is not** influenced by the output matching circuit $G_S = G_S(\Gamma_S)$
  - power gain added by the output matching circuit **is not** influenced by the input matching circuit $G_L = G_L(\Gamma_L)$
- Output /Input match can be designed **independently**
  - We can impose different demands for input/output
  - Total gain is:

$$G_T = G_S \cdot G_0 \cdot G_L \qquad G_T[dB] = G_S[dB] + G_0[dB] + G_L[dB]$$

Input matching circuit



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

- Maximum gain in the case of complex conjugate match

$$\Gamma_S = S_{11}^* \Rightarrow \qquad G_{S\,\max} = \frac{1}{1-|S_{11}|^2}$$

- For any other input matching circuit:

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} < G_{S\,\max} = \frac{1}{1-|S_{11}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

$$U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)} = 0.094$$

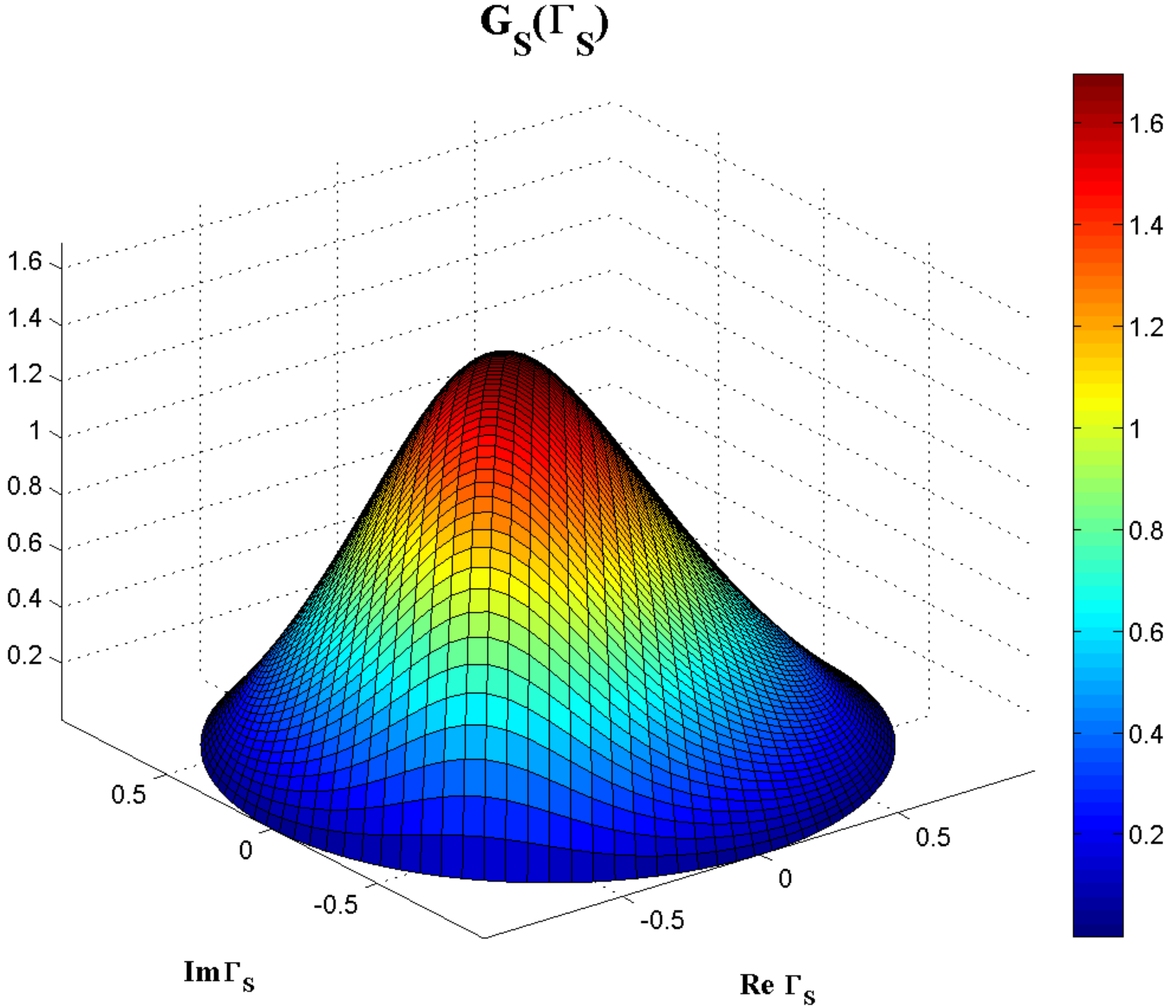
$$-0.783\,dB < G_T[dB] - G_{TU}[dB] < 0.861\,dB$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-|S_{22}|^2} = 17.83$$

$$G_{TU\max}[dB] = 12.511\,dB$$

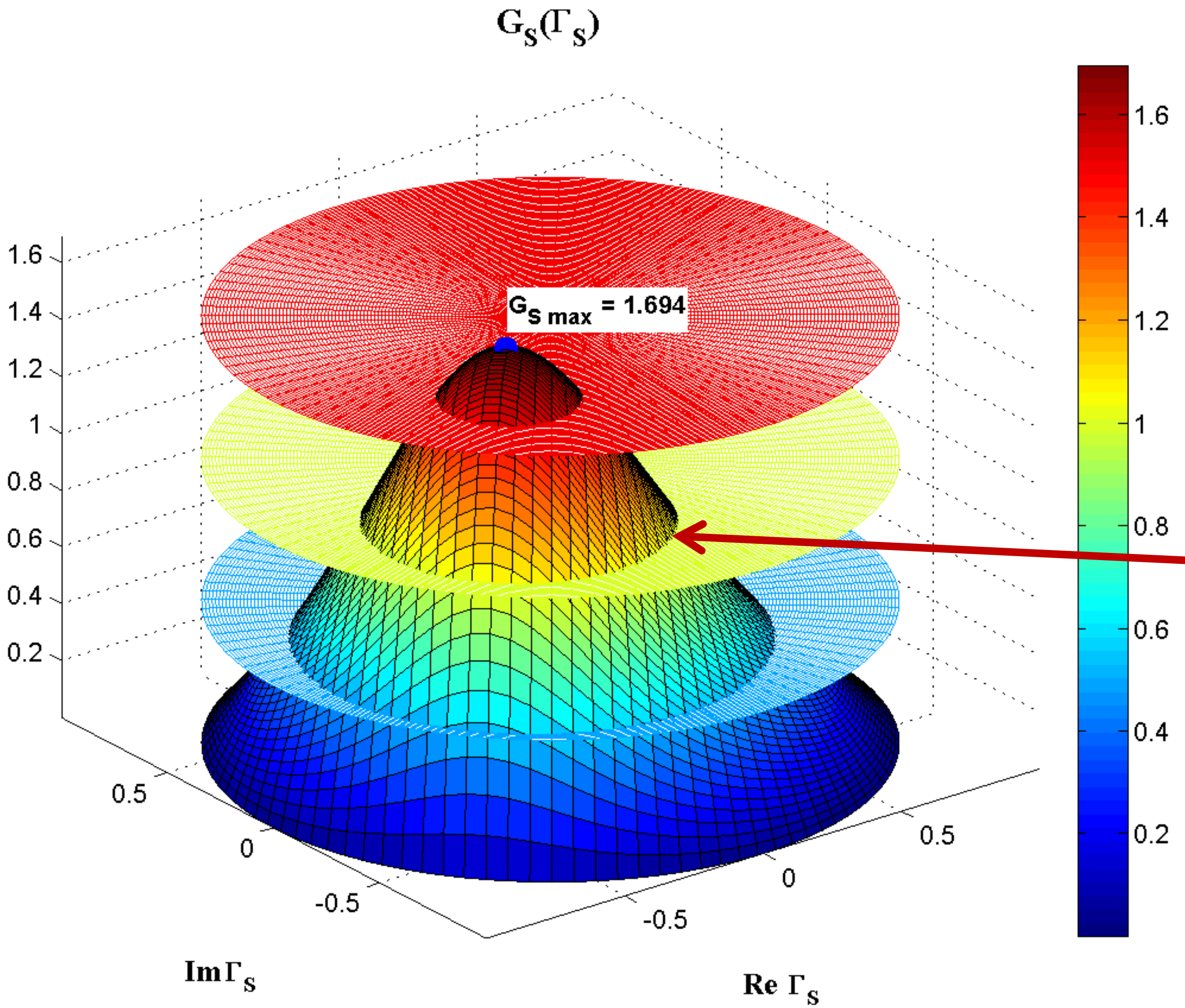
$$G_{S\max} = \frac{1}{1-|S_{11}|^2} = 1.694 = 2.289\,dB$$

$G_S(\Gamma_S)$



$$G_S = \frac{1-\left|\Gamma_S\right|^2}{\left|1-S_{11}\cdot\Gamma_S\right|^2}$$

$G_S(\Gamma_S)$, constant value contours



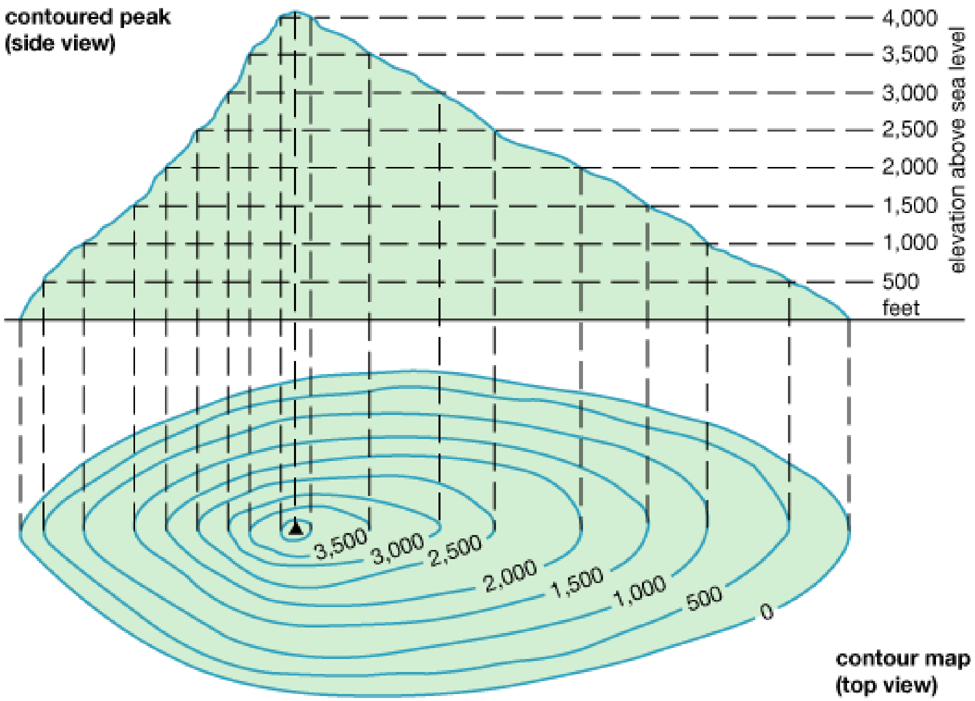
$G_S = 1.5$

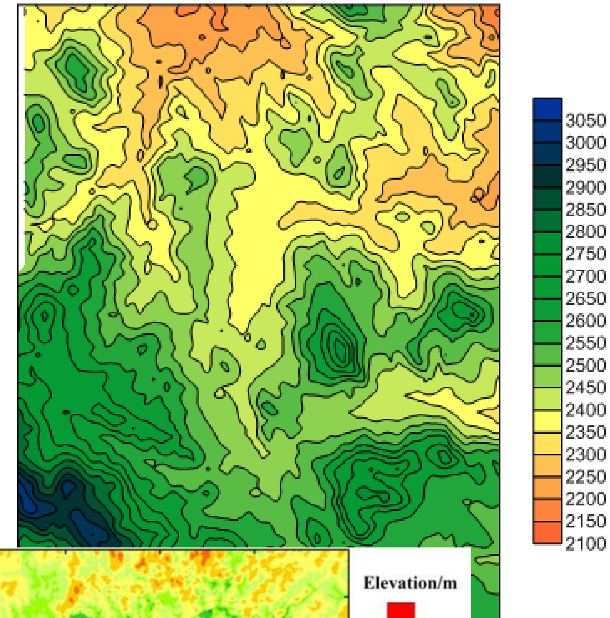
$G_S = 1.0$

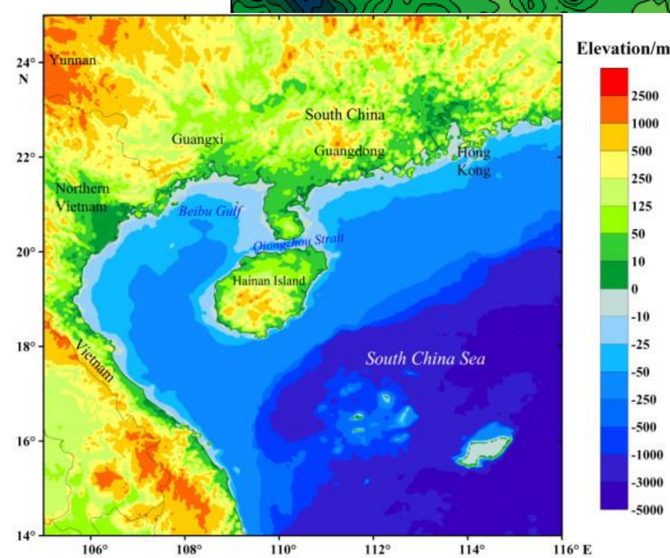
$G_S = 0.5$

Circles

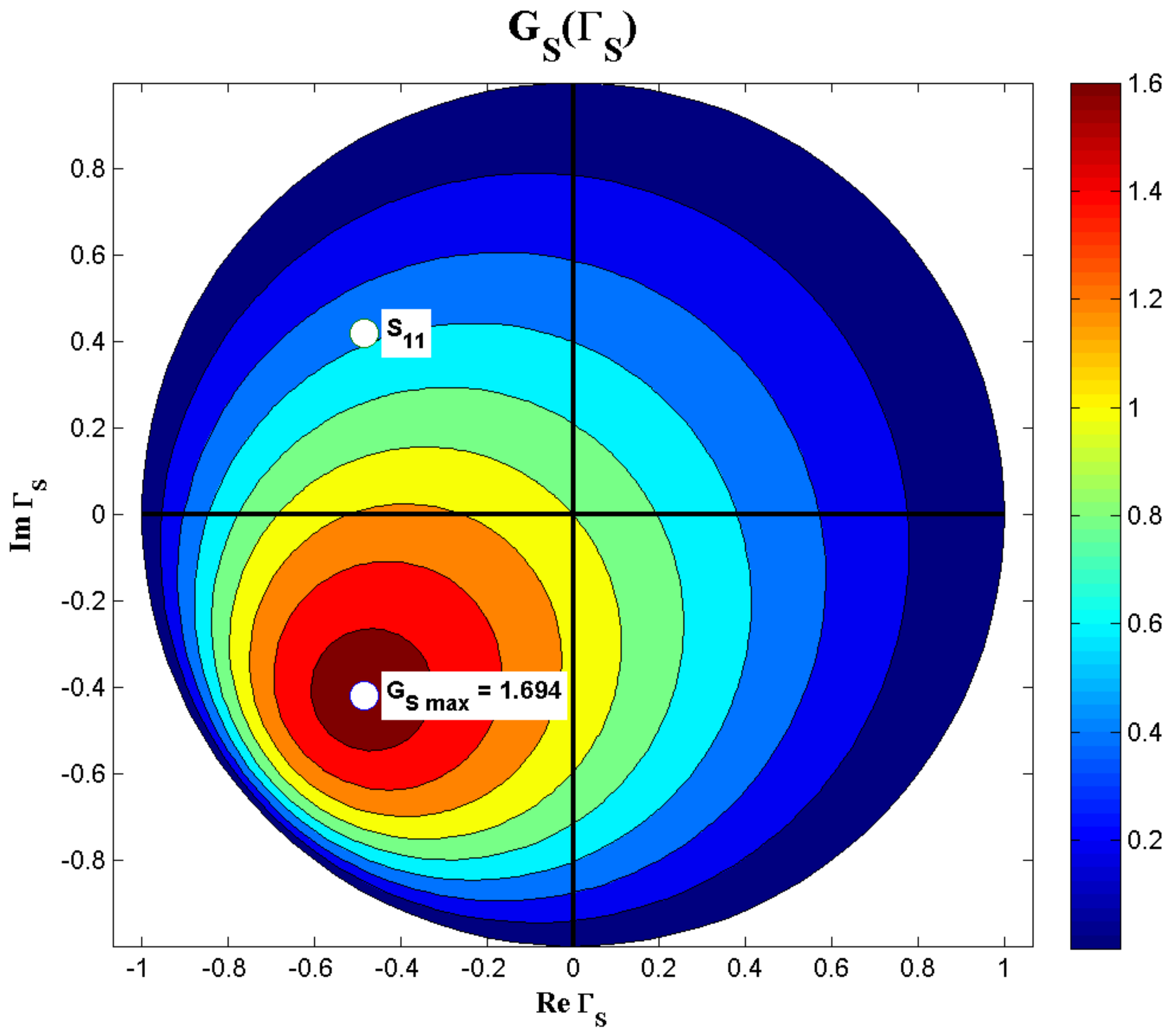
Contour map/lines







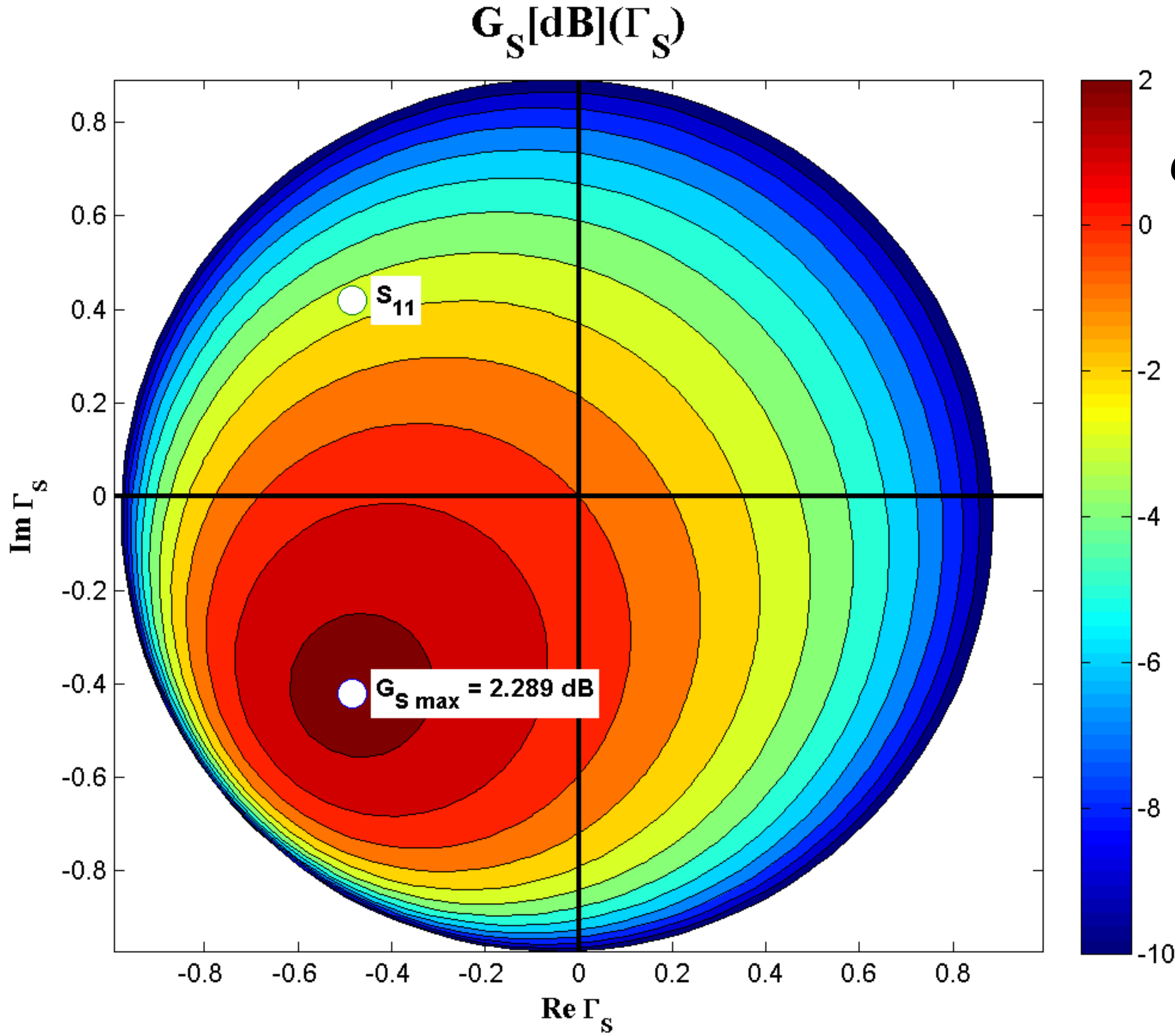
$G_S(\Gamma_S)$, constant value contours



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

$$G_{S\max} = G_S\Big|_{\Gamma_S = S_{11}^*}$$

$G_S[dB](\Gamma_S)$, constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\,max} = G_S\Big|_{\Gamma_S = S_{11}^*}$$

Input section constant gain circles

- The normalized gain factor (**linear scale!**)

$$g_S = \frac{G_S}{G_{S\max}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot\left(1-|S_{11}|^2\right) < 1$$

- Locus of the points with fixed values g_s<1

$$g_S\cdot|1-S_{11}\cdot\Gamma_S|^2 = \left(1-|\Gamma_S|^2\right)\cdot\left(1-|S_{11}|^2\right)$$

$$\left(g_S\cdot|S_{11}|^2+1-|S_{11}|^2\right)\cdot|\Gamma_S|^2 - g_S\cdot\left(S_{11}\cdot\Gamma_S+S_{11}^*\cdot\Gamma_S^*\right) = 1-|S_{11}|^2-g_S$$

$$\Gamma_S\cdot\Gamma_S^* - \frac{g_S\cdot\left(S_{11}\cdot\Gamma_S+S_{11}^*\cdot\Gamma_S^*\right)}{1-(1-g_S)\cdot|S_{11}|^2} = \frac{1-|S_{11}|^2-g_S}{1-(1-g_S)\cdot|S_{11}|^2} + \frac{g_S^2\cdot|S_{11}|^2}{\left[1-(1-g_S)\cdot|S_{11}|^2\right]^2}$$

$$|a+b|^2 = (a+b)\cdot(a+b)^* = (a+b)\cdot\left(a^*+b^*\right) = |a|^2+|b|^2+a^*\cdot b+a\cdot b^*$$

Input section constant gain circles

$$\left|\Gamma_S - \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2}\right| = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2} \qquad |\Gamma_S - C_S| = R_S$$

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2}$$

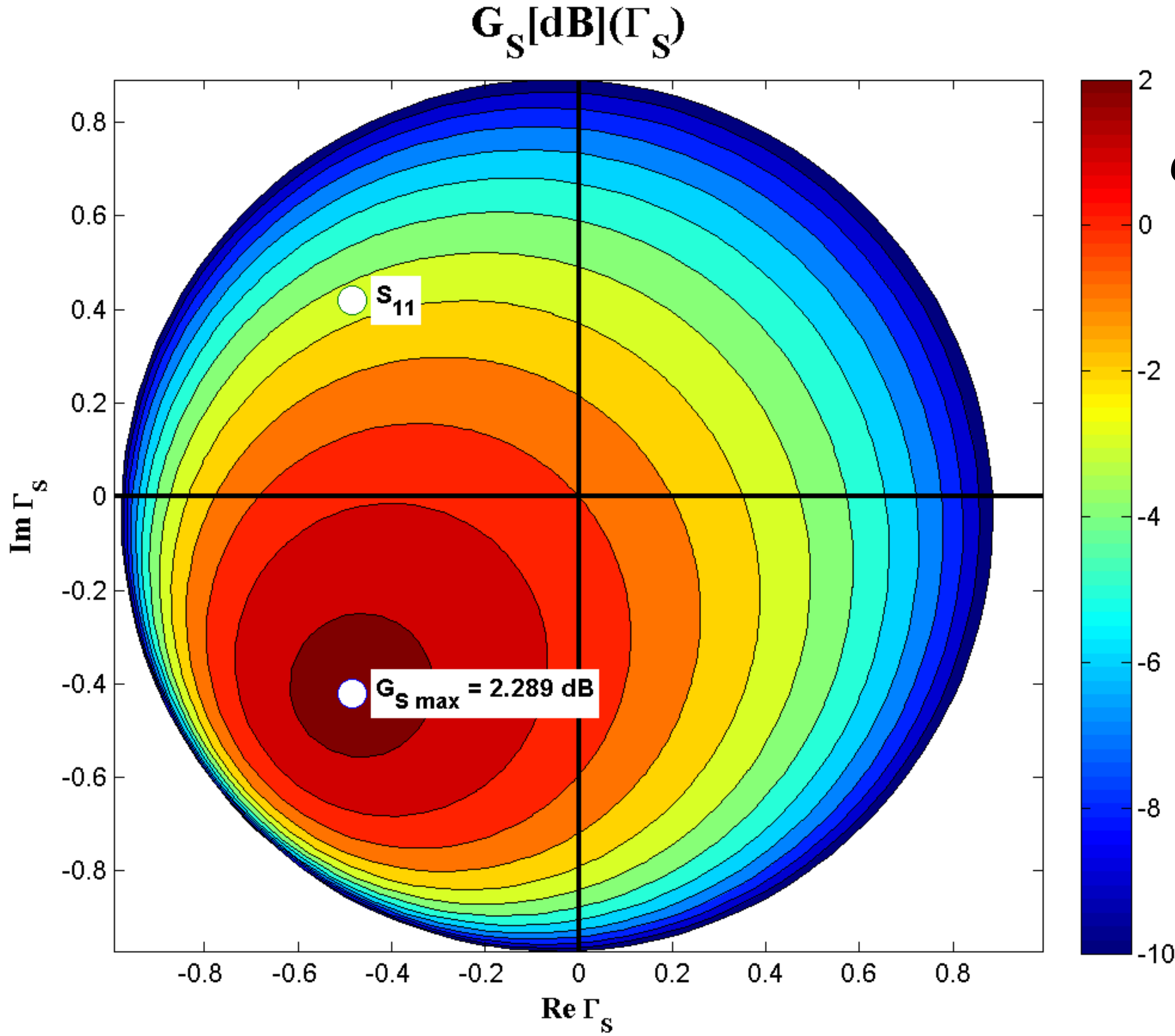
- Equation of a circle in the complex plane where Γ_S is plotted
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for $g_{circle} = G_{circle}/G_{Smax}$ will lead to a gain $G_S = G_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a gain $G_S < G_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a gain $G_S > G_{circle}$

Input section constant gain circles

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2}$$

- The centers of each family of circles lie along straight lines given by the angle of $\Gamma_{S\max} = S_{11}^*$
- Circles are plotted (traditionally, CAD) in **logarithmic scale** ([dB])
  - formulas are in **linear scale!**
- The circle for G_S = 0 dB will always pass through the origin of the complex plane (center of the Smith chart)

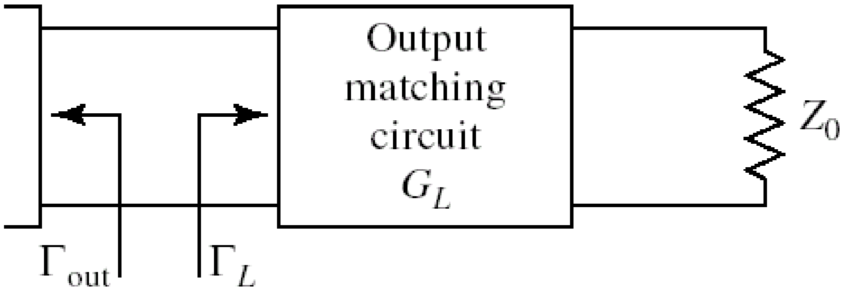
G_S[dB](Γ_S), constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*}$$

Output section constant gain circles



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

- Maximum gain for $\Gamma_L = S_{22}^* \Rightarrow$ $G_{L\max} = \dfrac{1}{1-|S_{22}|^2}$

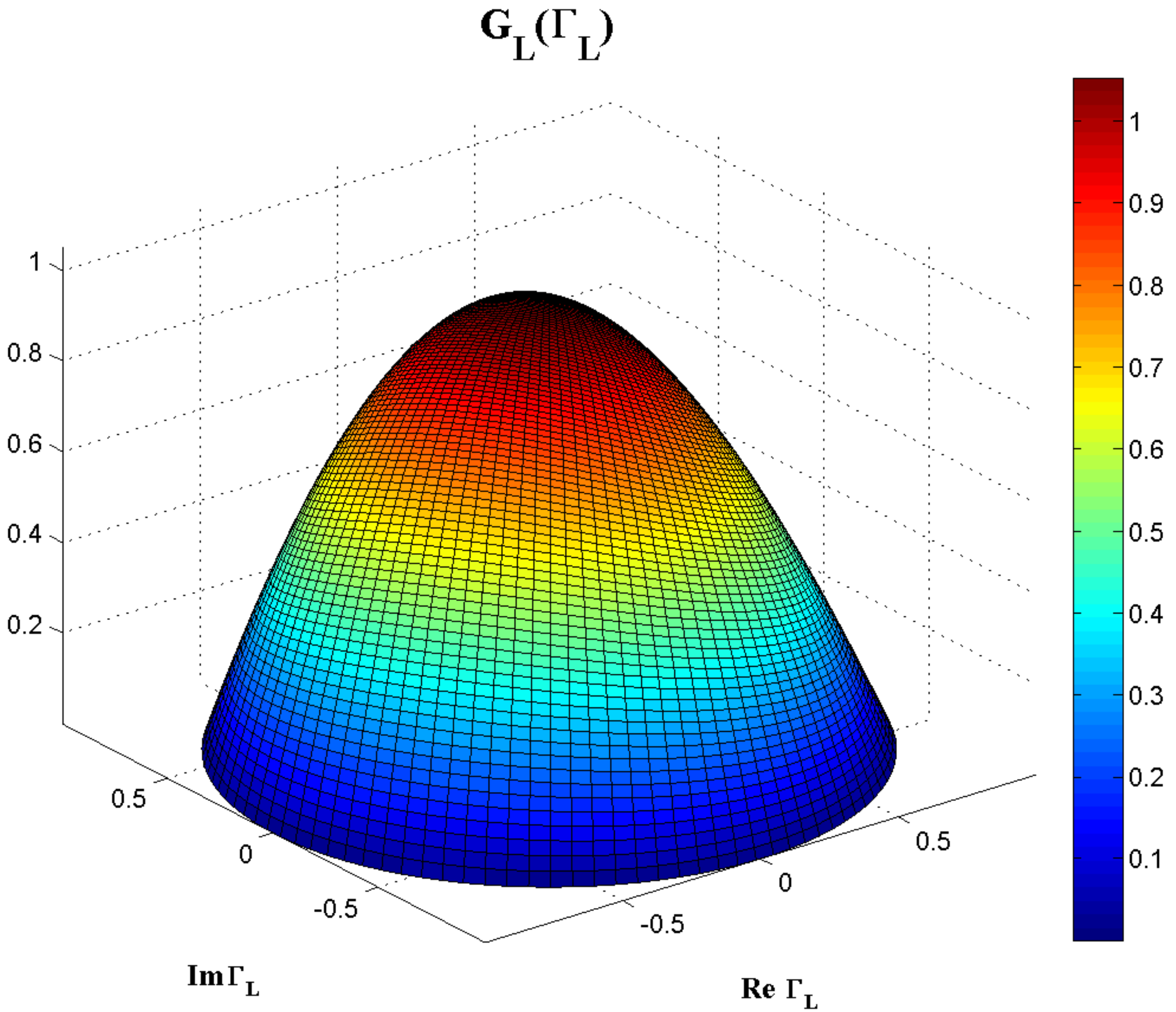
$$g_L = \frac{G_L}{G_{L\max}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}\cdot\left(1-|S_{22}|^2\right) < 1$$

- Similar computations

$$C_L = \frac{g_L\cdot S_{22}^*}{1-(1-g_L)\cdot|S_{22}|^2} \qquad R_L = \frac{\sqrt{1-g_L}\cdot\left(1-|S_{22}|^2\right)}{1-(1-g_L)\cdot|S_{22}|^2}$$

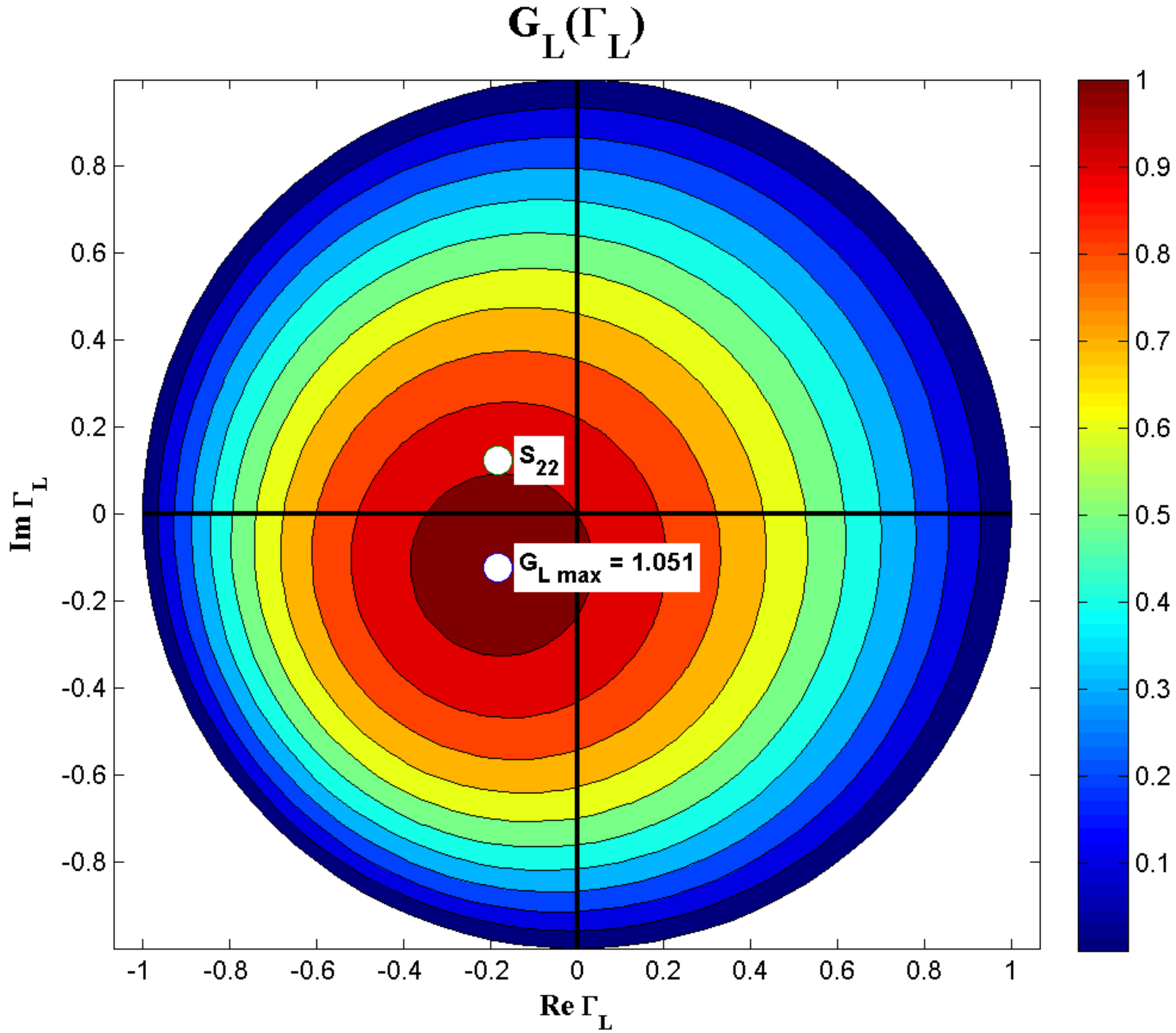
- Example $G_{L\max} = \dfrac{1}{1-|S_{22}|^2} = 1.051 = 0.215\ dB$

$G_L(\Gamma_L)$



$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

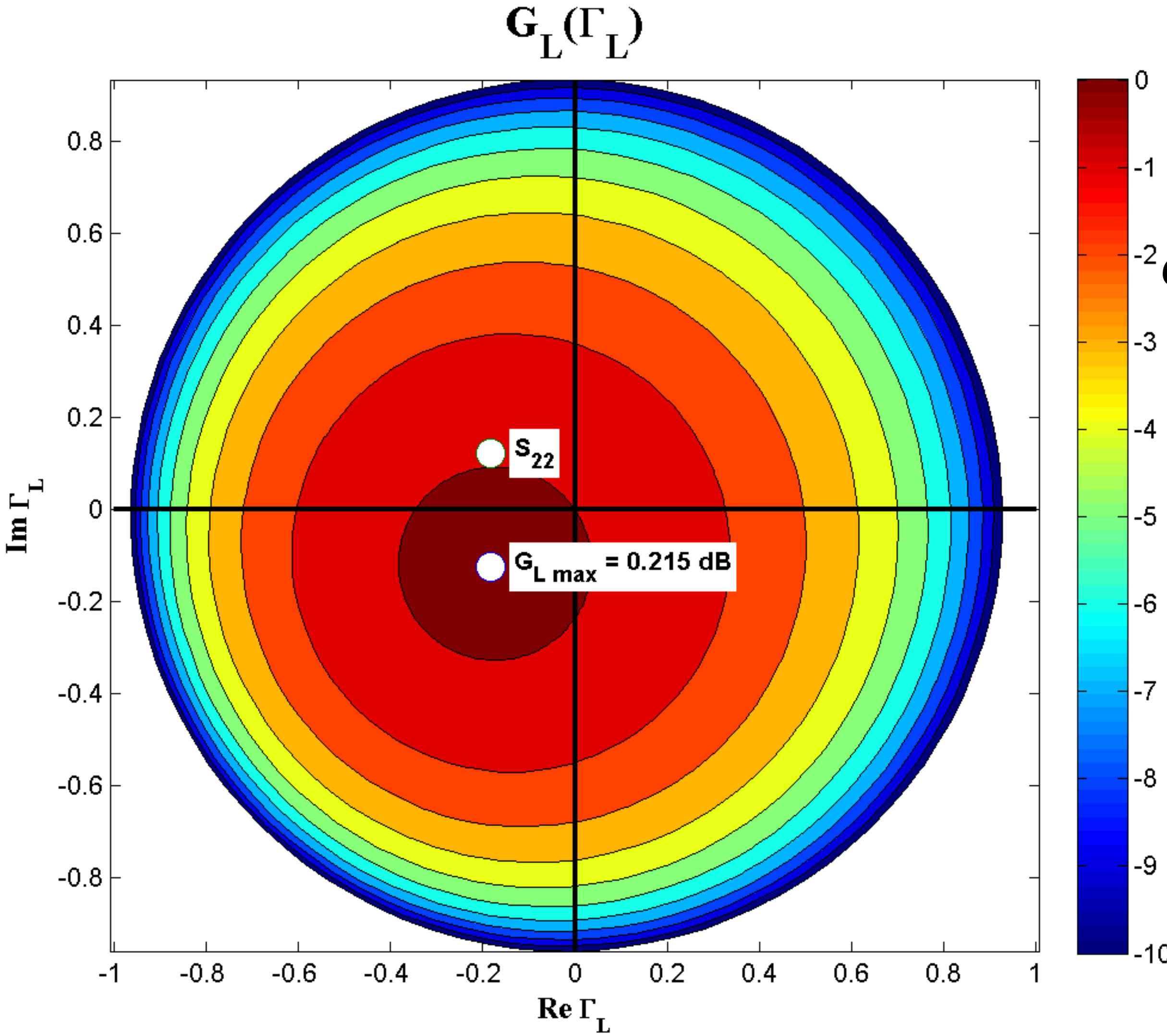
$G_L(\Gamma_L)$, constant value contours



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_{L\max} = G_L\big|_{\Gamma_L=S_{22}^*}$$

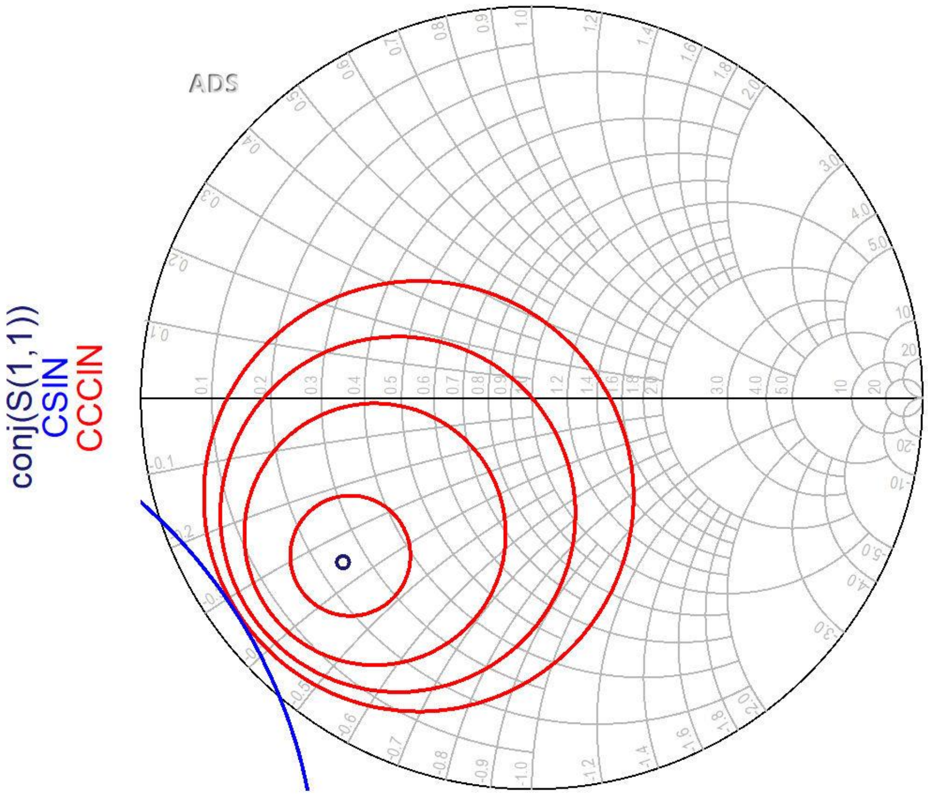
$G_L[dB](\Gamma_L)$, constant value contours

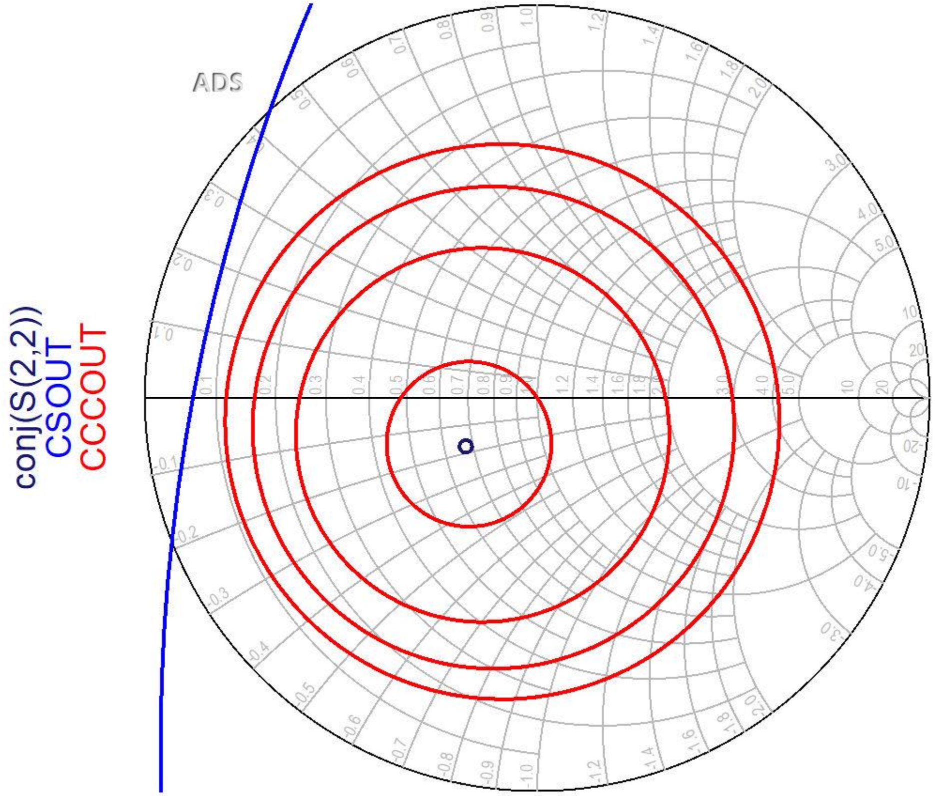


$$G_L[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}\right)$$

$$G_{L\max} = G_L\Big|_{\Gamma_L = S_{22}^*}$$

ADS

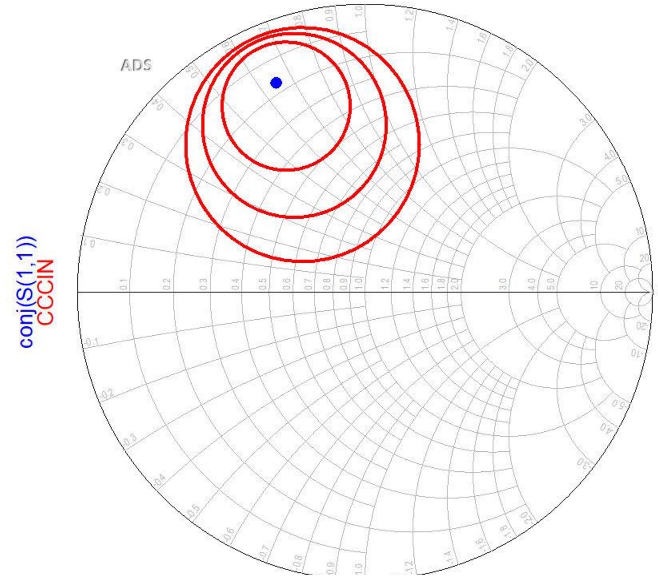


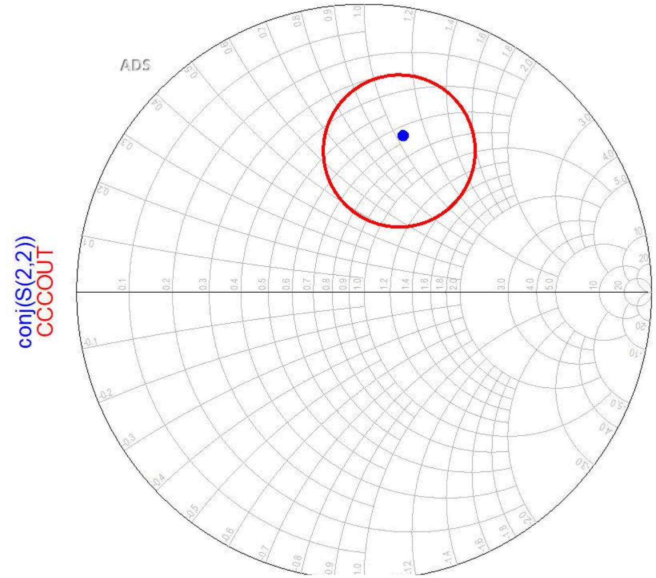


- Circles are plotted for requested values (**in dB!**)
- It is usefull to compute G_{Smax} and G_{Lmax} before
  - in order to request relevant circles

Schematic 1 – Lab 3

- **multiple circles (families)** are plotted and some required values are computed





Eqn gamma_opt=Sopt

Eqn G0=10*log(mag(S(2,1))**2)

Eqn GSmax=10*log(1/(1-mag(S(1,1))**2))

Eqn GLmax=10*log(1/(1-mag(S(2,2))**2))

freq	K	MAG	NFmin	Sopt	Rn	G0	GLmax	GSmax
5.000 GHz	0.533	15.296	0.700	0.660 / 106....	19.500	8.974	1.634	4.249

Design for Specified Gain

- We compute G_o, G_{Smax} , G_{Lmax}
- To obtain the design gain we **choose** supplemental gain needed (supplemental to constant G_o)
  - we account for the deviation that might arise from the unilateral assumption (using unilateral figure of merit U)

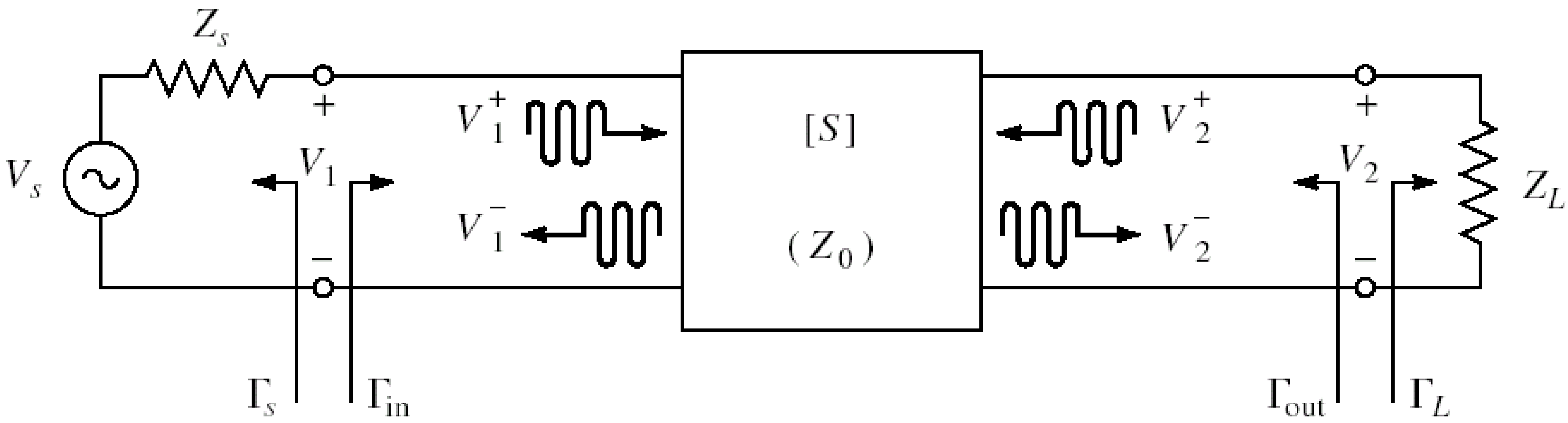
$$G_{design}[dB] = G_{S\_design}[dB] + G_0[dB] + G_{L\_design}[dB]$$

- We plot the circles for design (chosen) values G_{S_design} , G_{L_design}
- We design input and output matching circuits which move the reflection coefficient **on** or **inside** the design circles (depending on specific application requirements)

Microwave Amplifiers

Low-Noise Amplifier Design

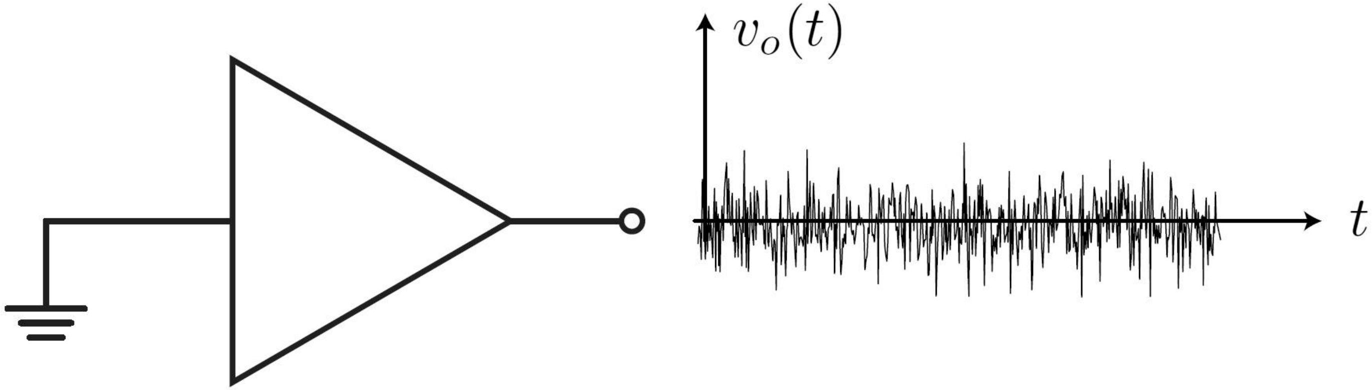
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - **noise** (sometimes – **small signals**)
  - linearity (sometimes – large signals)

Noise

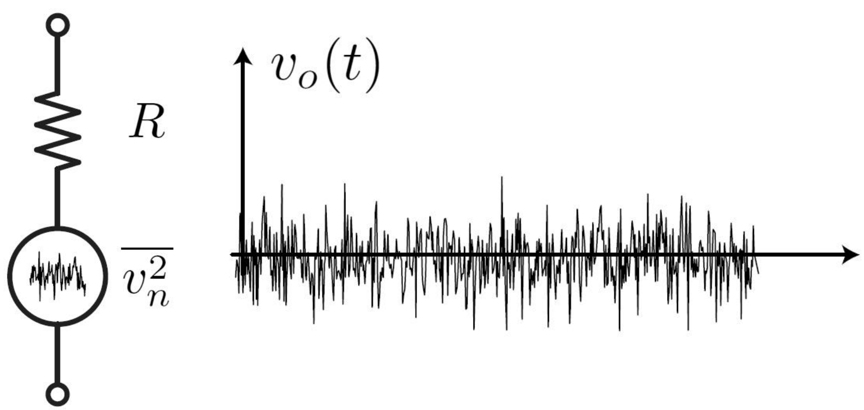
- Noise: random fluctuations of the signal

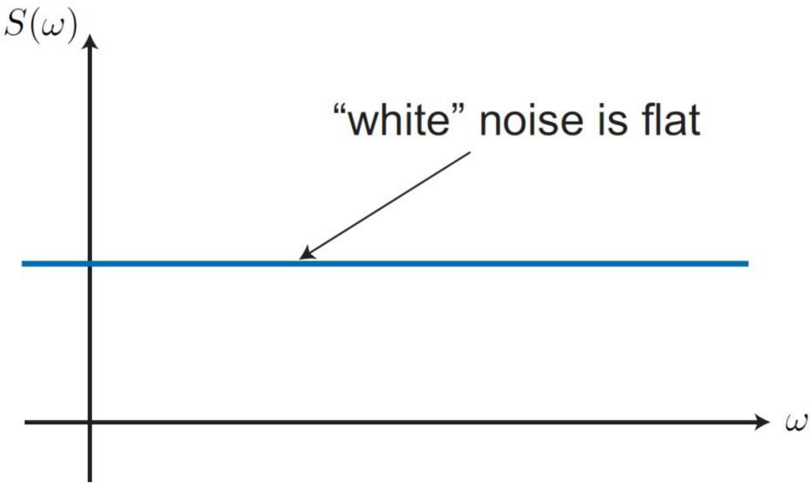


$$\overline{v_n(t)} = \langle v_n(t) \rangle = \frac{1}{T}\int_0^T v_n(t)\,dt = 0$$

$$\overline{v_n^2(t)} = \langle v_n^2(t) \rangle = \frac{1}{T}\int_0^T v_n^2(t)\,dt \neq 0 \qquad V_{n(ef)} = \sqrt{\overline{v_n^2(t)}}$$

Noise





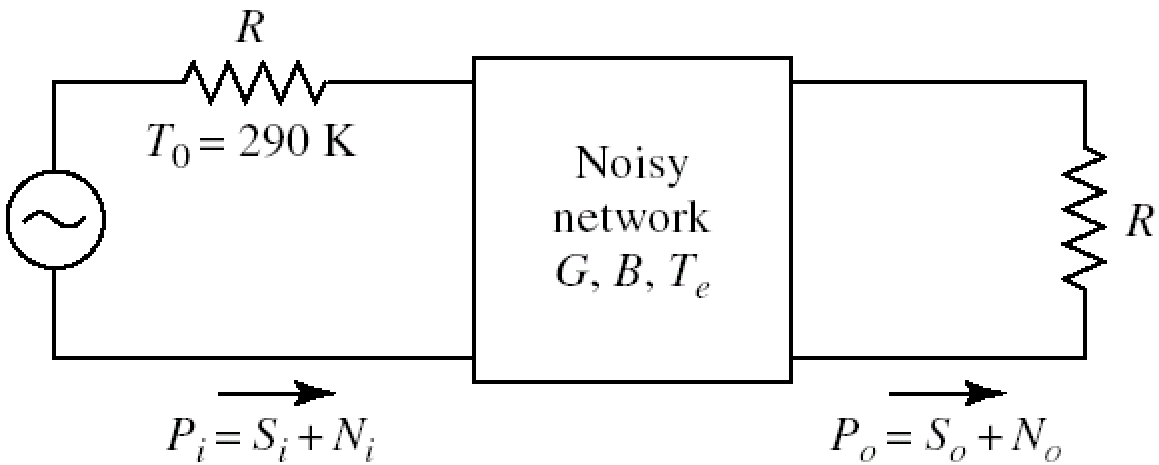
- effective noise voltage

$$V_{n(ef)} = \sqrt{4kTBR}$$

- noise power available (for maximum power transfer with impedance/resistance matching)

$$P_n = kTB$$

Noise Figure F



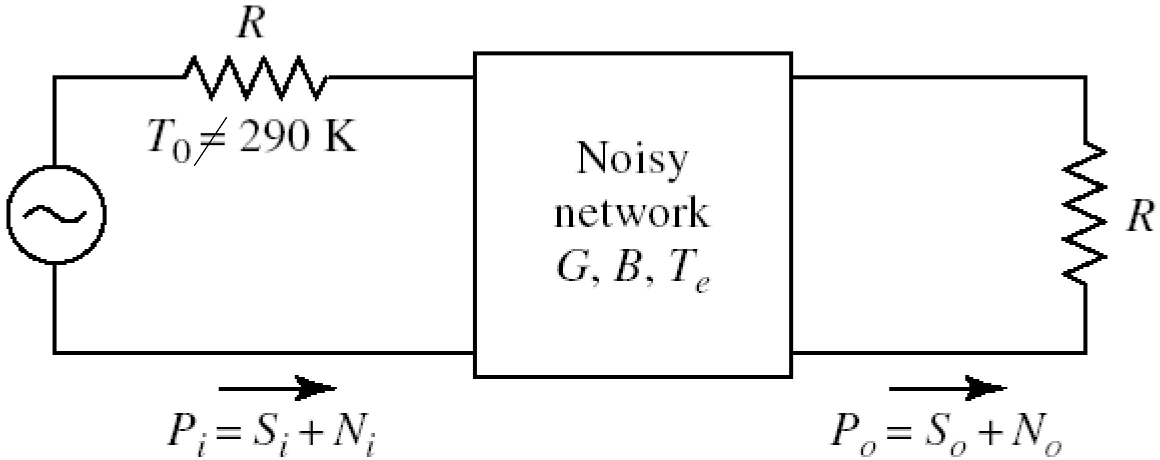
- The noise figure F, is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at To = 290 K (reference noise conditions)

$$F = \left. \frac{S_i / N_i}{S_o / N_o} \right|_{T_0 = 290K}$$

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

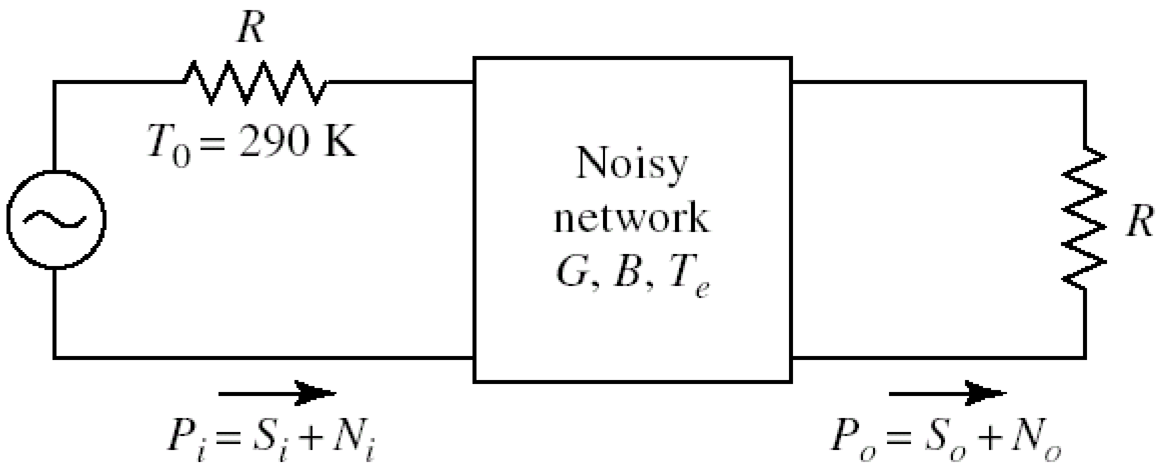
Noise Figure F



- The noise figure F, **is not** directly a measure of the reduction in signal-to-noise ratio between the input and output of a device, when the input noise power is different from that of the reference noise conditions

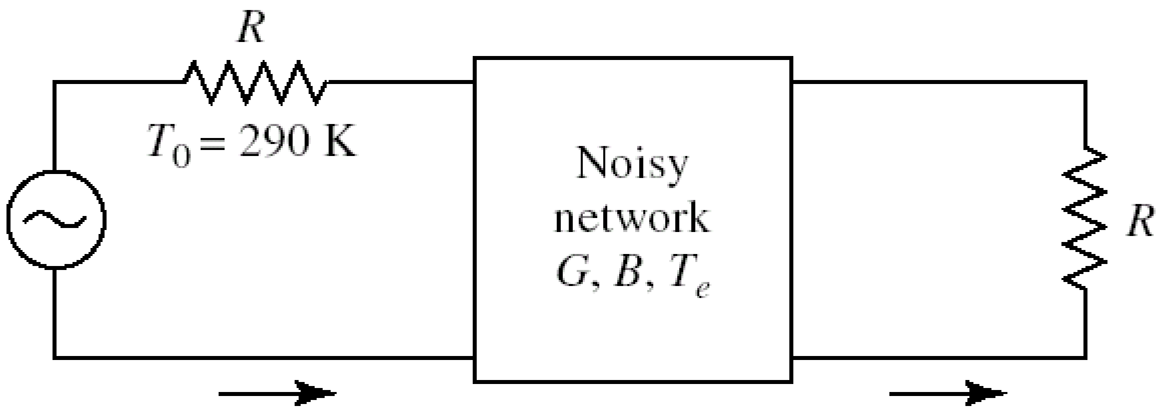
$$F \neq \left.\frac{S_i / N_i}{S_o / N_o}\right|_{T_0 \neq 290K}$$

Noise Figure F



- In general, the output noise power consists of two elements:
  - the input noise power amplified or attenuated by the device (for example amplified with the power gain G applied also to the desired signal)
  - a noise power generated internally by the network if the network is noisy (this power **does not** depend on the input noise power)

Noise Figure F



$P_1 = S_1 + N_1$ $\qquad$ $P_2 = S_2 + N_2$

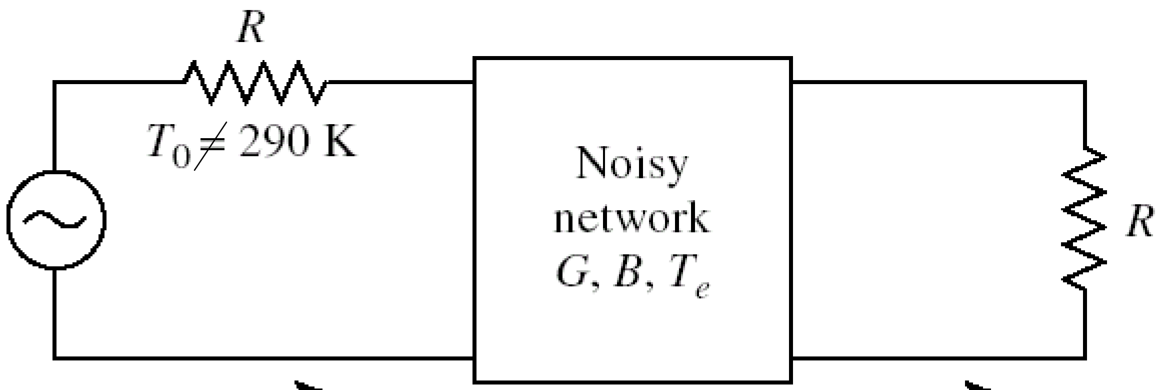
- Estimation of the internally generated noise power can be done using the Noise Figure F definition:

$$F = \left.\frac{S_1/N_1}{S_2/N_2}\right|_{T_0=290K,\, N_1=N_0}$$

$$N_2 = F \cdot N_0 \cdot \frac{S_2}{S_1} = F \cdot N_0 \cdot G$$

$$N_2 = N_0 \cdot G + (F-1) \cdot N_0 \cdot G$$

Noise Figure F



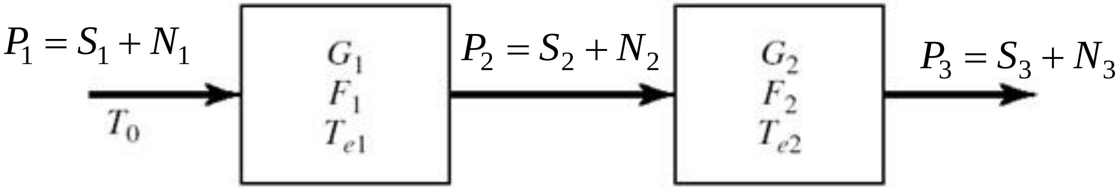
$$P_1 = S_1 + N_1 \qquad P_2 = S_2 + N_2$$

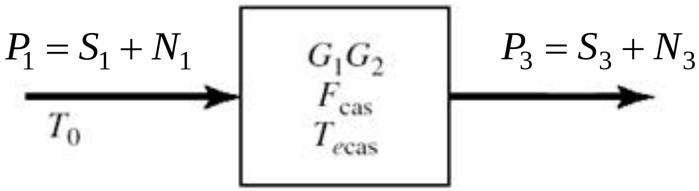
- We identify the two terms:
  - amplified input noise
  - internally generated noise
- When the input noise does not correspond to reference noise conditions (N1 ≠ No)
  - the internally generated noise does not change

$$N_2 = N_0 \cdot G + (F-1) \cdot N_0 \cdot G$$

$$N_2 = N_1 \cdot G + (F-1) \cdot N_0 \cdot G$$

Noise figure of a cascaded system





$$N_2 = N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1$$

$$N_3 = N_2 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2$$

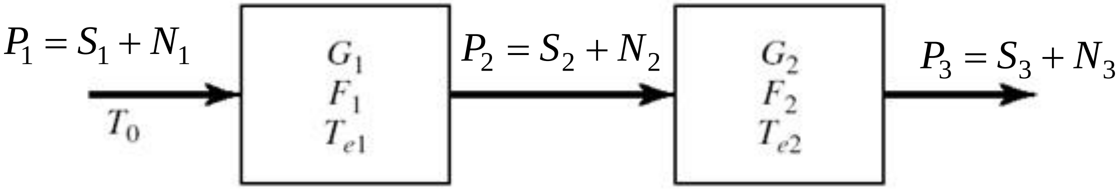
$$N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

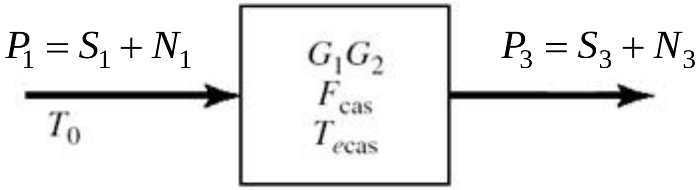
$$\downarrow$$

$$N_3 = [N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1] \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

Noise figure of a cascaded system





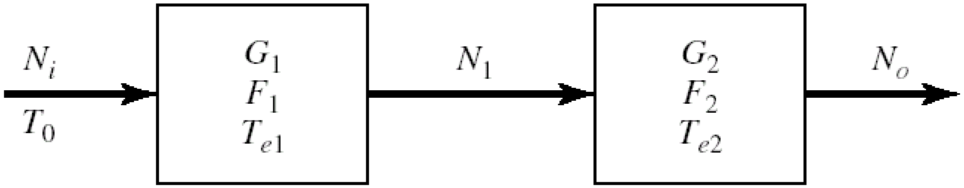
$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2 \qquad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

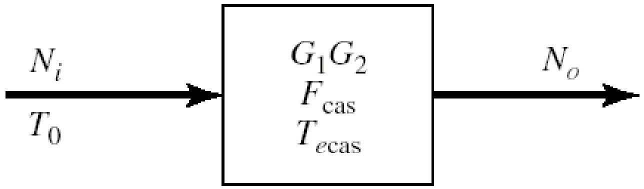
$$(F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 = (F_{cas} - 1) \cdot N_0 \cdot G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

Noise figure of a cascaded system



(a)



(b)

$$G_{cas} = G_1 \cdot G_2 \qquad F_{cas} = F_1 + \frac{1}{G_1}\left(F_2 - 1\right)$$

- Friis Formula (**!linear scale**)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Friis Formula shows that:
  - the overall noise figure of a cascaded system is largely determined by the noise characteristics of the first stage
  - the noise introduced by the following stages is reduced:
    - -1
    - division by G (usually G > 1)

Friis Formula (noise)

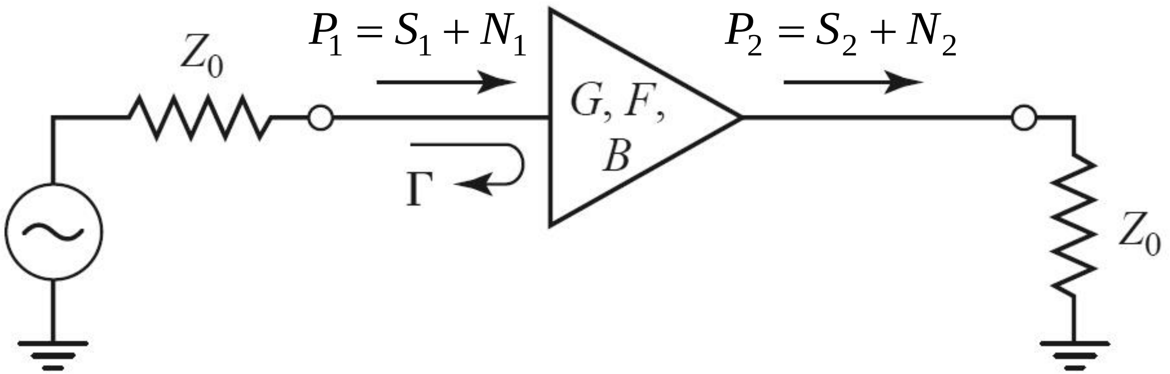
$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Effects of Friis Formula:
- in multi stage amplifiers:
  - it's essential that the first stage is as noiseless as possible even if that means sacrificing power gain
  - the following stages can be optimized for power gain
- in single stage amplifiers:
  - in the input matching circuit it's important to have noiseless elements (pure reactance, lossless lines)
  - output matching circuit has less influence on the noise (noise generated at this level appears when the desired signal has already been amplified by the transistor)

$$V_{n(ef)} = \sqrt{4kTBR} \qquad P_n = kTB$$

Noise Figure of a Mismatched Amplifier

- An input mismatched amplifier($\Gamma \neq 0$)



$$G_{ech} = G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + (F - 1) \cdot N_0 \cdot G = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + \frac{F - 1}{1 - |\Gamma|^2} \cdot N_0 \cdot G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G_{ech} + (F_{ech} - 1) \cdot N_0 \cdot G_{ech}$$

$$F_{ech} = 1 + \frac{F - 1}{1 - |\Gamma|^2} \geq F$$

- Good noise figure **requires** good impedance matching

Contact

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